

Update and extension of the 'Equivalent Slope' of speed changing level locomotion in humans: a computational model for shuttle running

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Summary statement: We present a revision/update of the 'Equivalent Slope' concept to estimate the metabolic cost of level locomotion at unsteady speed, application via a computational model to shuttle running.

Abstract

Controlled experimental protocols for metabolic cost assessment of speed changing locomotion are quite complex to be designed and managed. The use of the 'equivalent slope', i.e. the gradient locomotion at constant speed metabolically equivalent to a level progression in acceleration, proved to be useful to estimate the metabolic cost of speed changing gaits. However, its use with steep slopes forces to extrapolate the experimental cost vs. gradient function for constant running speed, resulting in less reliable estimates. The present study extended the model to work also with deceleration, and revised that predictive equation to be applied to much higher levels of speed change. The case of shuttle running at different distances (from 5+5 to 20+20m) was then investigated throughout the novel approach and software, and the predictions in terms of metabolic cost and efficiency well compare to the experimental data.

Introduction

Unsteady locomotion both in terms of nonlinear trajectory and speed changes is a common observation in everyday life of humans and animals, as well as in sport activities of bipeds and quadrupeds. The scientific interest regarding mechanical (maneuverability, static and dynamic stability, performance) and metabolic sustainability of those gait changes with respect to linear/constant speed locomotion has been met so far by a few papers (e.g. Alexander 2002, Wilson et al. 2018).

Moving at fluctuating speed in humans received some attention only in the last two decades. Particularly, the metabolic cost and mechanics of walking and running at imposed unsteady speed were studied (Minetti et al., 2001; 2013), although for small-intermediate accelerations/decelerations about the average speed. The interest about the metabolic implications of unsteady locomotion grew up further when trying to infer the players' effort in sports as soccer and rugby where locomotion during the match is far from occurring at constant speed. In addition, relevant speed changes are supposed to be associated to high metabolic cost, but the experimental protocol capable to reliably measure it collides with the problem of steady state condition. For this reason, an alternative urged to be found, and 'the equivalent slope' concept came at hand.

The concept of 'Equivalent Slope' (ES), has been mentioned so far in the biomechanics of cycling and of sprint running. In cycling, the "rolling resistance equivalent slope, a very shallow downhill gradient at which the negative potential energy changes balance the work necessary to overcome the rolling resistance" was suggested as a handy laboratory tool to estimate tyre friction on the ground (Ardigò et al., 2003). The concept was to convert the deceleration of coasting down (on the level) into a downhill slope at which the bicycle+rider could passively remain stationary on a treadmill (thus moving at constant speed). At that gradient, gravity provides a net forward component that equates the rolling resistance effect when moving on the level. Very conveniently, the coefficient of rolling resistance corresponds to the tangent of that slope angle.

In sprint running the equivalent slope (di Prampero et al., 2005) is an ingenious and convenient tool to infer the metabolic cost of transport of level running in (constant) acceleration by considering it an analogue of running uphill at a constant speed, for which the cost (C , J kg⁻¹ m⁻¹) is already available in the literature (Minetti et al., 2002):

$$C = 155.4 i^5 - 30.4 i^4 - 43.3 i^3 + 46.3 i^2 + 19.5 i + 3.6 \quad (1)$$

where i is the (downhill: negative, uphill: positive) gradient (in terms of the tangent of slope angle). This equation, designed to serve as an accurate description of the measured dataset, is valid only within $-0.45 < i < +0.45$, corresponding to the gradient range of metabolic experiments. As the Equivalent Slope for acceleration can be written as:

$$ES = \frac{a_f}{g} \quad (2)$$

where a_f (m s⁻²) is forward acceleration and g is gravity, the replacement of gradient i with ES in eq. 1 is safely allowed only when $-4.41 < a_f < +4.41$ m s⁻². Beyond these limits, the extrapolation of C can be unsafe (reliability decreases with the (squared) distance from the limit).

A recent paper (Zamparo et al., 2018) about mechanics and energetics of shuttle running, an activity incorporating large accelerations and even larger decelerations, and the fact that also sprint running involves accelerations (up to 7 m s⁻²) well beyond the indicated limits, both encouraged to update and further develop the previous predictive tools (di Prampero et al.,

2005; Minetti et al., 2002) in order: 1) to enhance inference reliability in an extended range of accelerations and 2) to make metabolic predictions also for decelerated running.

The extension to a wider range of acceleration benefits from recent metabolic results (Giovanelli et al., 2016) of uphill, constant speed running up to $i=+0.84$ that, according to eq. 2, corresponds to level accelerations up to 8.24 m s^{-2} .

The aim of the present study is to build a conceptual/computational framework allowing to infer the metabolic demand of accelerated/decelerated running, based either on the Equivalent Slope analogue (see above) or on the ultimate meaning of the cost of transport (see below), which is immediately applicable to 'shuttle runs' in humans, but is also potentially transferable to unsteady animal locomotion.

Mathematical Model

When pooling previous and recent results (Minetti et al., 2002; Giovanelli et al., 2016, see Fig. 1), the whole dataset confirms the trend to an oblique asymptote also at very steep gradients. This suggests to model the uphill part ($i>0$) of the phenomenon as a combined equation made of an oblique line (asymptote) summed to a decreasing exponential. The results, obtained by using a graphical/statistical package (Grapher, Apple Computers Inc., US) by using the uphill data only, is:

$$C_{PG} = 39.5 i + 3.6 e^{-4 i} \quad (3)$$

where C_{PG} stands for Cost of Positive Gradient (running).

Although there are no new data in the literature, to the author's knowledge, about metabolic cost of downhill running at $i<-0.45$, the visual inspection of Figure 1 suggests as reliable, so far, the trend shown by the oblique asymptote already indicated by the existing data (Minetti et al., 2002). By following the same approach leading to eq. 3 (the only difference is that here we need an increasing exponential), a new fitting of (just) the downhill data is obtained as:

$$C_{NG} = -8.34 i + 3.6 e^{13 i} \quad (4)$$

where C_{NG} stands for Cost of Negative Gradient (running). Regression coefficients in eqs. 3 and 4 have been slightly adjusted to reduce the number of digits and to assure continuity between the two curves (at $i=0$).

The accuracy of the newly proposed equation(s) for C has been tested by computing the ratio, in the range $-0.45 \leq i \leq +0.45$, between eqs. 4 and 3 with the negative and positive gradient branches of eq. 1, respectively, resulting in an average value of 0.9993. Curves for eqs. 3 and 4 are also plotted in Figure 1.

The present mathematical approach (eqs. 4 and 3) to data fitting is more modellistic than descriptive (eq. 1), and it comes from a suggestion about efficiency of locomotion introduced in the first half of the last century (Margaria, 1938). In synthesis, there has to be a minimum cost (C_{min} , J kg⁻¹ m⁻¹) for gradient running (and walking), which relates to coping with the inevitable work to sustain overall positive (uphill) and negative (downhill) changes in potential energy of the body (W_{vert} , J kg⁻¹ m⁻¹, where m_{vert} is the vertical meter travelled).

By following the formalism of Minetti et al. (2002),

$$C_{min} = \frac{W_{vert}}{eff} = \frac{g}{eff} \sin(\text{atan } i) \quad (5)$$

where *eff* refer to the efficiency of converting metabolic into mechanical energy. Depending on the sign of *i*, maximum *eff* value for positive work (0.25, $i > 0$) and for negative work (-1.25, $i < 0$) done by muscle force are used. This, together with the approximation $i = \sin(\text{atan } i)$ (see below), results in

$$C_{min} = 39.2 \, i \text{ (uphill gradients) and } C_{min} = -7.85 \, i \text{ (downhill gradients)} \quad (6)$$

whose coefficients closely resemble the linear components of eqs. 3 and 4 where, in accordance with muscle physiology (Woledge et al., 1985), the absolute ratio between negative and positive efficiencies (and, consequently, between slopes of the linear parts of C curves) is close to 5 (from eqs. 3 and 4, $\text{abs}(39.5/(-8.34))=4.7$).

The linear components of eqs. 3 and 4 are plotted in Figure 1 as straight lines.

The concept of Equivalent Slope of accelerated running has been introduced together with the Equivalent Mass (EM, di Prampero et al., 2005). EM, expressed as a fraction of the actual body mass, is a value greater than 1 to take into account the amount of the accelerating (or decelerating) vector in the analogous gradient (at ES) running at constant speed, and can be rewritten as:

$$EM = (ES^2 + 1)^{0.5} \quad (7)$$

In the original paper, the metabolic cost of accelerated running (C_{AR} , J kg⁻¹ m⁻¹) was computed by using equivalent versions of the present eqs. 2 and 7, in combination with eq. 1 where gradient i was replaced by ES, as:

$$C_{AR} = (155.4 ES^5 - 30.4 ES^4 - 43.3 ES^3 + 46.3 ES^2 + 19.5 ES + 3.6)EM \quad (8)$$

By adapting the same rationale to extended range of accelerations (and consequently, ES values), eqs. 2, 3 and 7 were combined as to obtain, after a little algebra:

$$C_{AR} = 0.102 (a_f^2 + 96.2)^{0.5} (4.03 a_f + 3.6 e^{-0.408 a_f}) \quad (9)$$

Eq. 9 can be safely use to predict C_{AR} for $0 < a_f < 8.24$ m s⁻².

The metabolic cost of decelerated running (C_{DR} , J kg⁻¹ m⁻¹) is obtained by considering

$$ES = \frac{d_f}{g} \quad (10)$$

where d_f (m s⁻²) is forward deceleration (a negative value), and by combining eqs. 10, 4 and 7, as:

$$C_{DR} = 0.102 (d_f^2 + 96.2)^{0.5} (-0.85 d_f + 3.6 e^{1.33 d_f}) \quad (11)$$

Equation 11 can be safely used for $-4.41 < d_f < 0$ m s⁻². As the fitting equation (eq. 4) is based on data already showing an asymptotic trend within the experimental gradient range, we are encouraged to use it with confidence even for $-8.24 < d_f < 0$ m s⁻².

Equations 9 and 11, alone or in combination, will allow to infer the metabolic cost of just accelerated running (as in sprints), in structured sequences of acceleration and deceleration bouts (shuttle running), and in sport activities where complex combinations of accelerated and decelerated running irregularly occur (as soccer, rugby, basketball, baseball, etc.).

A relevant comment is on the mechanical resemblance of the 'equivalent slope' when applied to different motion activities. When referring just the overall centre of mass, the mechanics of speed changing on a level can be converted into a constant speed at a given, equivalent, (uphill or downhill) slope. This is the case of the ES of bicycles (Ardigò et al., 2003), invoked to easily

estimate tyre rolling resistance, where the subject even refrains to pedal at the downhill gradient that makes the bike stationary on the treadmill (this is also a pure rolling resistance measurement as no air drag is acting on the subject).

When applying ES concept to running, which incorporates the use of metabolic data obtained when moving on gradients, a concern about other components of the total mechanical work could arise. The Internal Mechanical Work (W_{INT} , J kg⁻¹ m⁻¹), which accounts for the accelerations of body segments with respect to the body centre of mass (Cavagna and Kaneko, 1977), has been shown to be affected by many variables (Minetti, 1998):

$$W_{INT} = f v \left(1 + \left(\frac{d}{1-d} \right)^2 \right) q \quad (12)$$

where f is the stride frequency (Hz), v is progression speed (m s⁻¹), d is the duty factor (the fraction of the stride at which one foot is in contact with the ground), and q is a compound term incorporating anthropometric data of body segments.

It is possible that legged species as humans would choose different amounts of those variables, thus with an expected different W_{INT} , when moving in the two 'equivalent' conditions.

Particularly, inferring metabolism of level locomotion from gradient experiments where effort included the metabolic equivalent of a different (total) mechanical work could introduce some bias. Moving on slopes, particularly uphill running, is related to a much slower progression speed and a higher stride frequency (Minetti et al., 1994). Even the maintenance of the speed independency of metabolic cost of transport on slopes cannot help in this respect, and a check for W_{INT} should be done for the two compared conditions. This is the case for extreme accelerations (or decelerations) as during 100 m sprint, where ES is so high that speed and stride frequency (but also stride length) of a manageable metabolic experiment on gradient could potentially lead to an underestimation of W_{INT} .

Another important precaution is to consider all other determinants of the total mechanical work in the activity under investigation. Air drag, for example, can be relevant at the highest speed of the acceleration phase of sprint running (but not in ES for rolling resistance or slope running in the lab). When talking about shuttle running, rather, equations 9 and 11 can take care of the metabolic equivalent of the accelerative and decelerative phases, respectively, but the energy required to rotate the body at speed inversion is not included in the prediction.

This new mathematical model, based on the revamped concept of ‘equivalent slope’, can be implemented in new activity logger aimed to detect and monitor daily and physical activity with an improved analysis on the energy expenditure. Different from the present computational scheme, where acceleration and deceleration phases in shuttle running has been modelled as exponential functions of time, activity monitors (with GPS) would start from the continuous daily recording of body geolocation, from which instantaneous speed and acceleration would be obtained and fed into the described model (eqs. 9 and 11).

Computational Model of Shuttle Running

The model is based on the assumption that the accelerative and decelerative phases of a maximal Shuttle Run (SR) of different leg distances are portions of the same patterns exhibited when performing a very long shuttle run (say 20+20m). In a short distance SR (for instance a 10+10m SR) speed will reach a lower maximum value than in a longer SR, but its raise from zero and descent to zero (in a single leg) will follow the same exponential pattern of the much long-lasting acceleration and deceleration of the longest SR. This tendency also comes from a recent biomechanical analysis of SR from 5+5 to 20+20m (Zamparo et al., 2018), and, just for the acceleration phase, also from 20m sprint experiments on the same subjects (to allow comparison, 20+20m SR shows accelerative phases of about 13m).

The dynamics of speed change for acceleration and deceleration can be modelled according to a mono exponential function (e.g. Furusawa et al., 1927, di Prampero et al., 2005):

$$v = v_{max} \left(1 - e^{-\frac{t}{\tau}}\right) \quad (13)$$

where v_{max} is the asymptotic, maximum speed (m s^{-1}), and τ is the time constant (s). Equation 13 is valid for

SR acceleration phases when $t \geq 0, \tau_{af} > 0$, and for

SR deceleration phases when $t \leq 0, \tau_{df} < 0$.

In the computational model, it seemed useful to represent on the time axis a deceleration phase first ($t < 0$), followed by the acceleration ($t \geq 0$), as to represent in the same graphic frame (one leg of) SR of very different lengths (see Fig. 2 legend) and to accommodate the iterative process to determine the maximum speed for each of them ($\hat{v}_{max,SR}$).

The computational model has, as inputs, data from long distance SR (say 20+20m): v_{max} , τ_{af} and τ_{df} . These 3 parameters were obtained as follows: v_{max} and τ_{af} came as unknowns of a non linear regression of eq. 13, fitting speed data in acceleration phase, while τ_{df} was the only unknown parameter of a non linear regression of eq. 13 where v_{max} was imposed (as one of the results from the other regression), fitting just deceleration data (see Appendix A2).

Time course of distance, speed, deceleration/acceleration, ES, EM and C are based on the new equations developed in this paper, as explained in Figure 2 legend. Then, depending on whichever SR distance of interest (s_{SR} , actually just one leg, 10m for, say, 10+10m SR) an iterative process finds the relevant average C, the average apparent efficiency of muscle positive work, and other outcomes as the maximum speed reached.

The computational process starts from the (obvious) fact that the maximum speed reached at the end of the acceleration phase is the same of the start of the deceleration phase. The tentative value for $\hat{v}_{max,SR}$ starts from v_{max} and decreases it by small amounts; for each of them (which in Figure 2 corresponds to lower the horizontal dashed lines): 1) deceleration and acceleration durations are calculated (as intersections with the 2 exponential curves for speed, and marked by two vertical dashed lines in Figure 2), 2) deceleration and acceleration distances are obtained (as values on the s curves corresponding to the two identified time intervals), 3) and summed to obtain the total inferred SR distance (single leg), 4) which is compared to the s_{SR} by using a small threshold; 5) if the estimated distance is higher than the goal, the tentative value for $v_{max,SR}$ is lowered and a new iteration starts.

At the end of the process the average metabolic cost of that SR (C_{SR} , J kg⁻¹ m⁻¹), due just to speed changes of the body centre of mass, can be obtained by the Δs -weighed mean of the C curve within the iteratively 'established' time frame of that specific SR (i.e. within each pair of vertical dashed lines in Fig. 2). The entire computational process was designed by using Labview Programming Language (vers. 13, National Instruments, Austin, US); the software ran on a MacBook Pro (Apple, Cupertino, US) laptop computer. Also, early stages of the mathematical model were tested by using Grapher (Apple, Cupertino, US).

The software algorithm also provides an estimate of the average 'apparent' efficiency of the positive mechanical work during each SR, based on the ES values during SR. Level running (at constant speed) is associated to equal amount of positive and negative work of the body centre of mass (due to equal excursions of potential and kinetic increases and decreases). That 'external' work could be called 'apparent' as some of the positive (negative) work is not generated by muscles: it comes 'at no metabolic cost' from mechanical strain energy released by (the amount previously stored, again at no metabolic cost, in) tendons. When running on uphill (downhill) slopes, positive (negative) work becomes predominant, and beyond the slope range of ± 0.35 gradient running shows a monotonic increase (decrease) of the body centre of mass (thus just positive (negative) work is done), which impairs the chances of storing (releasing) elastic energy in (from) tendons (Minetti et al., 1994).

Therefore, with respect to the slope, the apparent efficiency of positive mechanical work is supposed to be: a) 0 for downhill slope steeper than -0.35 , b) an increasing value up to 0.80 at $i=0$ (in that range and in the next, different mix of positive and negative work are performed), which is the maximum apparent efficiency of positive muscle work in level running at the maximum speeds of SR (Cavagna and Kaneko, 1977), c) a decreasing value down to 0.25 (maximum muscle efficiency) at a slope $=+0.35$, d) beyond which 0.25 would be constant as only positive work is done (see Figure 3).

In Figure 2 the time course of apparent efficiency (eff , which depends on ES) is shown, within the relevant time frame, as a thick white curve. From the calculated average value along one SR leg (eff_{mean} , supposedly the same for the other leg) we could infer how much mechanical energy saving via elastic mechanism has occurred (zero for $eff_{mean} \approx 0.25$ and the maximum possible when approaching $0.75-0.80$).

Predictions of average C_{SR} (on level) rely, as mentioned, on gradient running (Minetti et al., 1994; Giovanelli et al., 2016) where W_{INT} was the result of different values of the crucial variables involved (see eq. 12). Thus, estimates of C_{SR} based on the ES concept have to be corrected by adding the metabolic equivalent of the 'extra' W_{INT} that would fill the gap toward the actual W_{INT} of SR events. To do this, we compared single stride W_{INT} data obtained during a 20m sprint (Pavei et al., in preparation) to the average values measured in steady running experiments at different gradients (Minetti et al., 1994). To allow a functionally meaningful estimation of the W_{INT} gap between the two conditions, data measured on slopes where time-

aligned according to when, during the sprint, ES value was close to the investigated uphill gradient.

Also, the metabolic energy of body turning at the end of SR legs is not included in the model based on eqs. 9 and 11. Here three approaches are available: a) estimating the minimum mechanical and metabolic cost to twist the body 180° about the vertical axis in the same time interval of SR experiments, b) chasing literature findings of the metabolic cost of running in circles of small radius, and c) inspecting studies about the metabolic cost of changing direction in running.

a) the twist of the body can be modelled as a turning from angle (θ , rad) 0 to π about the vertical axis, lasting a time Δt (s), with an initial and final angular speed (ω , rad s⁻¹) equal to zero and a time course following a sine function of time:

$$\omega = A \sin (B t) \quad (14)$$

where A is the peak speed, occurring at $\theta = \pi/2$, equal to $\pi^2/2$, and $B = \pi/\Delta t$. From this equation the angle (θ , rad) time course:

$$\theta = \frac{A}{B} [1 - \cos (B t)] \quad (15)$$

and the angular acceleration (α , rad s⁻²):

$$\alpha = A B \cos (B t) \quad (16)$$

can be calculated (by integration and differentiation, respectively, of Eq. 14). Since rotational power (\dot{W}_{rot} , W) is defined as

$$\dot{W}_{rot} = I_z \alpha \omega \quad (17)$$

where I_z is the moment of inertia (kg m²) of the body, the positive mechanical work of (half) a turn (W_{turn} , J) can be obtained as:

$$W_{turn} = \int_0^{\frac{\Delta t}{2}} \dot{W}_{rot} dt \quad (18)$$

which, after calculus and a little algebra, results as:

$$W_{turn} = \frac{\pi^4}{8} I_z \quad (19)$$

Finally, to incorporate this component into the overall metabolic cost of SR, namely as $\text{J kg}^{-1} \text{m}^{-1}$, the eq. 19 turns into:

$$C_{turn} = \frac{\pi^4}{8 m \text{eff}_{muscle} SRleg_{dist}} I_z \quad (20)$$

where m is body mass (kg), eff_{muscle} is muscle efficiency for positive work (0.25-0.30) and $SRleg_{dist}$ is the distance of a single SR leg. When considering $I_z = 3.83 \text{ kg m}^2$ and $m = 75 \text{ kg}$, eq. 20 becomes:

$$C_{turn} = \frac{2.49}{SRleg_{dist}} \quad (21)$$

This estimate of C_{turn} has to be considered the minimum cost of a 180° turning of the (slightly crouched) body about its vertical axis.

b) studies about the energy cost of running along small circles are rather scanty in literature. To the authors' knowledge the smallest investigated turn radius is 1 m (Minetti et al., 2011). The metabolic cost of running along that circle was found to be almost speed independent, and equal to $283.1 \pm 64.1 \text{ mL O}_2 \text{ kg}^{-1} \text{ km}^{-1}$, or $5.78 \pm 1.31 \text{ J kg}^{-1} \text{ m}^{-1}$. It has to be reminded that 1 m radius refers to the distance of the pivot from where feet were placed; actually the 'effective' turn of the body centre of mass occurred at a radius of 0.72-0.84 m. From those data, the distance for half a circle was 3.14 m, thus:

$$C_{turn} = \frac{18.1}{SRleg_{dist}} \quad (22)$$

c) a paper on the metabolic cost of changing direction in running helps to add an estimate of the extra metabolic cost with respect to the predictions from eqs. 9 and 11. Zamparo and colleagues (2014) showed that shuttles with full inversion (180°), when compared to other with just the straight (0°) stop-and-go, resulted (non-significantly, though) in about $+0.9 \text{ J kg}^{-1} \text{ m}^{-1}$ metabolic cost, in shuttle runs of 10 m (corresponding to about $9.0 \text{ J kg}^{-1} \text{ turn}^{-1}$). Thus, the extra cost is here:

$$C_{turn} = \frac{9.0}{SRleg_{dist}} \quad (23)$$

expressed in $\text{J kg}^{-1} \text{ m}^{-1}$.

Results

Figure 4 shows the difference in Mechanical Internal Work when considering data from SR (at maximal performance) and from uphill running (at submaximal constant speed) at the same ES/slope. The comparison points out about $1 \text{ J kg}^{-1} \text{ m}^{-1}$ of additional internal work (0.925 ± 0.069) that has to be added, in the form of its metabolic equivalent

$$C_{extraWint} = 3.083 \quad (24)$$

$\text{J kg}^{-1} \text{ m}^{-1}$, for a muscle efficiency of 0.30, to the C_{SR} estimate obtained from eqs. 9 and 11.

Input parameters for the computational model were: $v_{max} = 7.175 \text{ m s}^{-1}$, $\tau_{af} = 0.803 \text{ s}^{-1}$ and $\tau_{df} = 0.472 \text{ s}^{-1}$ (data source for the quoted regressions is Zamparo et al., 2018, maximal 20+20m SR only); their CV ranges from 7.7 to 10.9 %.

Model-estimated maximum speed of SR ($\hat{v}_{max,SR}$) was compared to (the average value of) measured maximum speed, at the different SR distances, and resulted in an overestimation of $4.96 \pm 1.64 \%$.

Figure 5 shows experimental results of C_{SR} as (Buglione and di Prampero 2013; Zamparo et al., 2015) and the predictions from the present model for each SR distance. Input parameters for the algorithm were $SRleg_{dist}$ and the measured v_{max} , τ_{af} and τ_{df} from kinematic motion capture of 20+20m SR experiments (Zamparo et al., 2018), as mentioned. Here, inferred C_{SR} from the comprehensive model is

$$C_{SR} = C_{AR} + C_{DR} + C_{turn} + C_{extraWint} \quad (25)$$

with C_{turn} taken as an average of eqs. 22 and 23,

The amount of C_{AR} and C_{DR} , i.e. the main determinants of C_{SR} , depends on the prevalence of high values of (positive or negative, respectively) ES during the SR leg. As short distance SRs are more travelled at high acceleration and deceleration than long distance SRs, average ES is high. It has to be mentioned, though, that within the time course of each SR leg, very short distances are travelled at the end of the braking phase and at the beginning of the accelerative phase. Thus, despite of the high ES values of those phases, the influence of their related high

cost on average C_{AR} and C_{DR} (units are J per kg and per metre travelled) is mitigated by the distance-based weighed mean performed along each phase time axis.

When simulated acceleration/deceleration patterns from the measured average v_{max} , τ_{af} and τ_{df} and their associated ES values were fed into the relationship between slope and efficiency shown in Figure 3, the average apparent efficiency of positive work (by assuming a fixed ratio $eff-/eff+$ of 5) was found to be 35.3, 29.9, 23.2 and 17.5% for 20+20, 15+15, 10+10 and 5+5m shuttles, respectively. Those values are shown together with the ones experimentally obtained in Figure 6.

Discussion

The proposed model, which includes a revision of previous equations and an extension to decelerations of the 'Equivalent Slope for speed changing running' (di Prampero et al., 2005), together with the cost of the eventually additional Internal Work and of body turning, closely predicts (dashed curve in Fig. 5) the metabolic cost of Shuttle Running at different distances (Buglione and di Prampero 2013; Zamparo et al., 2015; Zamparo et al., 2018), a motor activity with bouts of maximal increases/decreases of running speed. Although the general trend of experimental C in SR is paralleled by the new ES predictions, the overall underestimation of just considering the cost of speed changes suggests to consider, for each specific motor act, all the potential sources of metabolic extra cost. As already argued above, there is a subtle bias potentially embedded in ES metaphor: (steady) running C at each slope (which is almost speed independent) incorporates the metabolic cost of a mechanical internal work that depends on a submaximal stride frequency. When trying to infer C for sprint running or, as here, in maximal SR, the W_{INT} increase due to a much higher stride frequency has to be considered. In the proposed case, the cost of turning has been estimated but there are other metabolic components, as isometric contractions and stabilizing co-contractions, that were not. These could be the reasons for the underestimation of C_{SR} at the shortest leg distance.

In addition to the metabolic cost prediction, the model combines ES concept with biomechanical findings about the mix of positive/negative work in running at different gradients (Minetti et al., 1994) and predicts values of apparent efficiency of muscle positive work, at the different SR lengths, which closely resembles (see Fig. 6) the measured values (Zamparo et al., 2018). Experimental data and predictions suggest that SR shorter than 15+15m

do not exploit the elastic energy storage/release typical of the 'landing-take off sequence' operated by tendons, as witnessed by efficiency values compatible with muscle activity only (as already found in 5+5 m SR by Zamparo et al., 2016). Recently it has been proposed, based on a combination of experiments and modelling, that elastic energy stored in tendons at the beginning of a maximal sprint is remarkable (Lai et al., 2016). This stored energy, differently from coming from an otherwise wasted potential and kinetic energy of the body centre of mass in the typical running bounce, derives from part of the positive work done by muscle contraction to explosively accelerate the body in the first few steps in sprinting. It is likely that in this case muscle power enhancement, rather than the muscle work saving, is the main goal and it is achieved through 'muscle power amplification' (Galantis and Woledge 2003). Due to the central role and the (net) amount of muscle positive work in the early stages of sprinting, we expect that the decreased apparent efficiency at short SR (caused by the prevalence of high ES of the initial strides), which is part of our results, remains compatible with those recent findings.

Limitations due to extrapolation of eq. 8 at very high slopes (or ES) have been improved by eq. 9, and eq. 11 now completes the prediction range by including negative gradients (or ES); this allows to safely infer the metabolic cost of running acceleration and deceleration for a speed change from -8.24 m s^{-2} to 8.24 m s^{-2} , corresponding to ES from -84 to +84%. Although the upper limit is very close to the maximum acceleration ability of humans, this is not true for the lower limit: previous papers dealing with the maximum negative power during drop landing (Minetti et al., 1998) and with kinetic analysis of speed changes during maximal SRs of different lengths (Zamparo et al., 2018) show that, as expected from the Force/Velocity relationship of muscle contraction, eccentric performance is much higher than concentric one also in in-vivo, complex motor acts. Thus, as Giovanelli and co-authors (2016) extended the metabolic cost measurements of running to much higher uphill gradients than previously (from +45% to +84%), the same should be done for downhill running at (equivalent) slopes up to almost double ($\times 1.7$, in absolute terms) the actual uphill limit (namely from -45% to -143%).

Approximations in the mathematical and computational model can affect the overall predictions. Although the new equations for C (eqs. 3 and 4) very closely fit experimental data of gradient running (Minetti et al., 2002, Giovanelli et al., 2016), their asymptotic trend, particularly for steep downhill slopes, can improve by investigating a wider gradient range (as mentioned) and the theoretically non-linear relationship between 'unavoidable' metabolic cost and gradient (see Appendix A1).

Inset graph in Figure 2 reveals that the assumption ruling the simulation, i.e. that short distance SRs use 'truncated' portions of acceleration and deceleration phases displayed at the maximum investigated distance, generates speed time courses with a sharp peak in between. This slightly differs from the experimental patterns (Zamparo et al., 2018). As the inaccuracy pertains to a zone where small accelerations and decelerations (and the related ES values) occur, we are confident that a potential correction (spline) would not alter the conclusion from the obtained predictions.

By inspecting the newly obtained C curve for acceleration and the corresponding ES (thick blue and orange in Fig. 2, respectively), a much simpler rule of thumb for replacing eq. 9 emerges as:

$$C_{AR} = \frac{a_f}{0.29} + 3.6 \quad (26)$$

Eq. 26 can be safely used when estimating the metabolic cost of accelerations in the range $0.80 < a_f < 8.24 \text{ m s}^{-2}$ (or $0.08 < \text{ES} < 0.84$). This comes with no surprise because, as mentioned previously (Minetti et al., 2013), the units of mechanical (and metabolic) cost of transport, $\text{J kg}^{-1} \text{ m}^{-1}$, correspond to m s^{-2} . Thus, at ES corresponding to accelerated running where only positive work is involved, the C_{AR} is simply obtained by dividing a_f by a customary efficiency value (0.29, here optimized to get average C_{AR} very close to predictions of eq. 9 for all SR distances) to obtain its metabolic equivalent, then summed to the cost of level running (i.e. $3.6 \text{ J kg}^{-1} \text{ m}^{-1}$).

Unfortunately, no easy rule of thumb for decelerated running allows to replace eq. 11, mostly due to the complex mix between positive and negative mechanical works, and their efficiencies, occurring at negative slopes/ES (Minetti et al., 1994).

The proposed rule of thumb for the cost of accelerated running can come as a handy tool when literature data on metabolic cost of constant speed running at different gradients is not available, as in animal locomotion. For each specific species, values 0.29 and 3.6 in eq. 26 should be replaced by the relevant 'efficiency of positive muscle work' (0.25 if unknown) and by the average cost of transport for bouncing gaits (trot or gallop) on level at constant speed, respectively. For instance, eq. 26 for accelerated locomotion in horses (data from Minetti et al., 1999), could become

$$C_{AR} = \frac{a_f}{0.25} + 2.2 \quad (27)$$

Differently from SR, where eq. 26 successfully predicted the cost of accelerations, eq. 27 is a suggestion to be tested in future metabolic experiments on horses.

As shown in this paper and in the previous literature (di Prampero et al., 2005; Osgnach et al., 2010; Gaudino et al., 2013; Minetti et al., 2013; Gaudino et al., 2014; Coutts et al., 2015; Kempton et al., 2015; di Prampero et al., 2015), the 'Equivalent Slope' concept can be profitably used in estimating metabolic cost of speed changing running. Other than shedding lights in the metabolic effects of remarkable speed oscillation in a gait where inherent velocity changes occur even at constant speed, the present revised methodology has potentially wide applications in activity monitored daily life (submaximal speed changes) and exercise physiology, particularly in sport activities where unsteady locomotion is prevalent (submaximal and maximal speed changes). Also, the time course of metabolic power (= instantaneous cost x instantaneous speed) can be calculated starting from one of the proposed computation frameworks (the 'rule of thumb' equations) to work out the maximum metabolic performance required in prey/predator settings. The next step would be the development of a model for speed changing walking, with predictions compared to the experimental measurements published so far in the literature.

Appendices

A1) note about the statistical model equation

The proposed regression equation (eqs. 3 and 4) incorporates a linear function of gradient i (see below) and an exponential component. The former, as mentioned, accounts for the inspection trends in Figure 1 at steep gradients and their role in representing the minimum gravitational work that has to be done. The latter has been conceived to represent the deviation of C from the two linear components at gradients in the range $-0.30 < i < +0.30$, where C likely reflects also the metabolic equivalent of other mechanical determinants as the cost of a mixed positive and negative work, which tends to disappear outside that gradient range (Minetti et al., 1994).

The approximation $i = \sin(\arctan i)$ holds only in the range $-0.45 < i < +0.45$, with a tendency of eq. 5 to deviate even consistently (-16% at $i = +0.80$) from eq. 4. For the aims of this paper, devoted to improve the predictive efficacy of eq. 1 at even steeper uphill slopes and to extend the 'Equivalent Slope' model to the estimation of metabolic cost of running decelerations, the suggested linearized regression $C = a i + b e^{c i}$ can be regarded as an approximate model incorporating part of the effects of the minimum gravitational work that has to be done. We leave the deepening of understanding about the discrepancy between observed C at very steep gradients and eq. 5 to future investigations.

A2) note about the statistical analysis of SR kinematic

The 3D coordinates of each body segment, sampled at 100 Hz by a 35-camera system (Vicon Oxford Metrics) in a recent study on SR kinematics (Zamparo et al., 2018) allowed to obtain the displacement speed of the body centre of mass. Speed data from acceleration and deceleration phases of just the first leg in maximal 20+20m SR were analysed. The acceleration phase was fitted according to eq. 13 by means of a non-linear regression model where both v_{max} and τ were estimated. The deceleration phase was fitted according to the same equation, by means of a non-linear regression model where v_{max} was imposed (as a result from the previous regression) and τ was estimated. This statistical strategy was adopted on one hand to better capture the experimental trend of running speed to reach the maximum value 'in acceleration', on the other to avoid speed discontinuity of SR leg pattern as reconstructed starting from eqs. 4 and 3. Such a granted continuity later allowed the computational algorithm to iteratively find, for each SR distance, the correct acceleration and deceleration timing compatible with that distance (see Computational Model of Shuttle Running).

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Competing interest

The authors declare no conflict of interests

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Figures

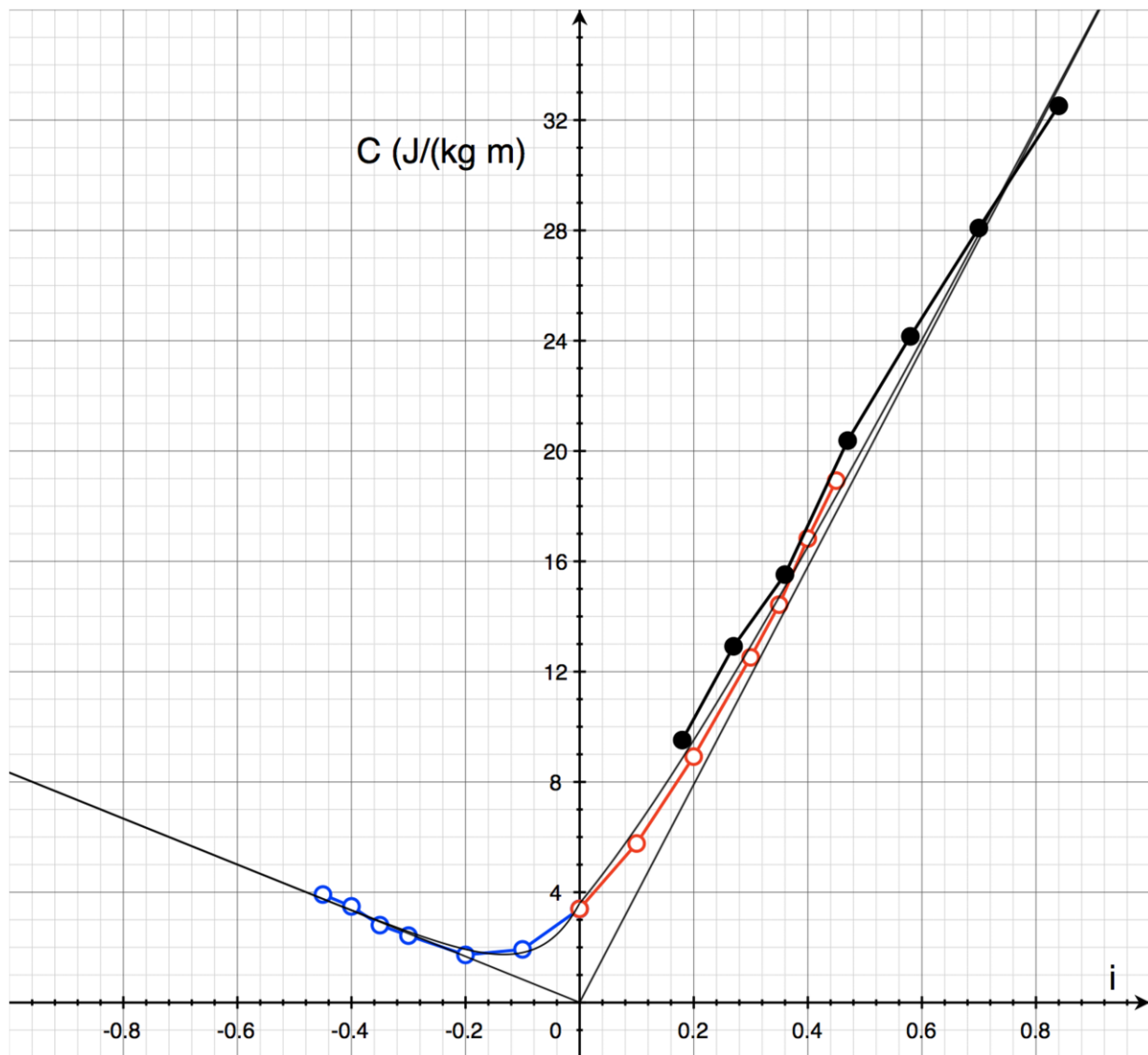


Figure 1: Metabolic cost of running (C , $\text{J kg}^{-1} \text{m}^{-1}$) vs. gradient (i). Open circles refer to Minetti et al. data (2002): positive gradient in blue, negative in red. Solid black circles refer to Giovanelli et al. (2016) data. The thin black line across circles represents curve for equation 3 and 4 (see text), whereas the linear (asymptotic) component of equation 3 and 4 are presented as straight black lines starting from the axes origin.

Cost estimation of Shuttle Run through Equivalent Slope

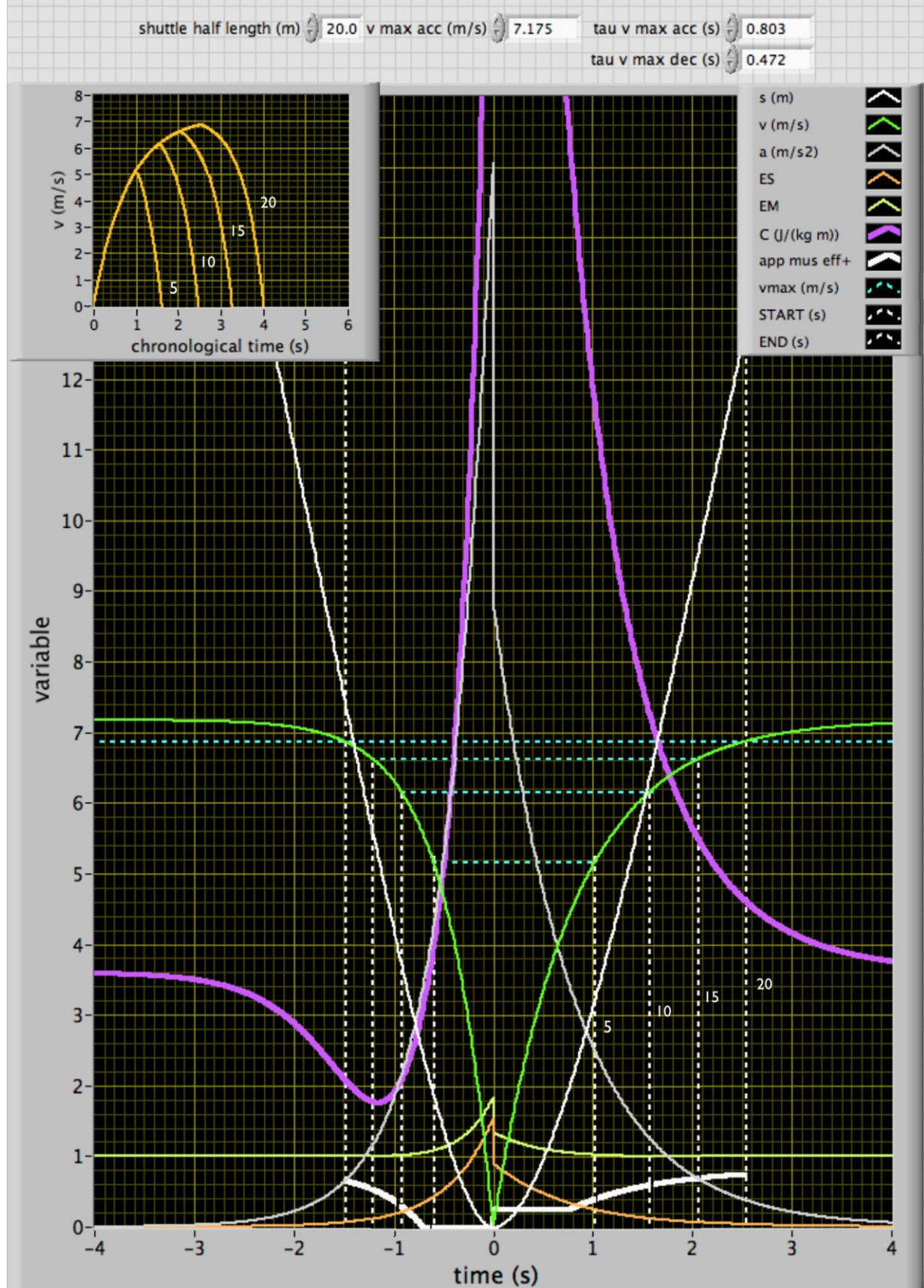


Figure 2: The output of the computational version of the present model applied to the prediction of Energy Expenditure during Shuttle Runs at maximal speed and different distances is shown. In the main graph, the exponential raise and decrease of speed (v , m s^{-1} , green curve), set by the maximum speed and the two tau (τ) coefficients, together with the shuttle total distance, are the actual inputs to the program. Deceleration has been drawn before the acceleration phase as to represent just one leg of the shuttle and as a strategy to accommodate, in the same graph, (half) shuttle runs of different lengths. Curves for space travelled (s , m , white curve) and acceleration (a , m s^{-2} , grey curve) have been obtained by integration and differentiation, respectively. As a matter of graphical convenience, s and a have changed sign when time is negative (i.e. during decelerative phase). Curves for ES (orange), EM (light green) and C ($\text{J kg}^{-1} \text{m}^{-1}$, purple) have been computed according to equations (2, 10), 7, (9, 11), respectively. ES for the deceleration phase has changed sign for graphical purposes. For each SR leg distances (20+20m, 15+15m, 10+10m, 5+5m) the horizontal dashed line represents the maximum speed chosen from the iterative procedure devoted to make the sum of the deceleration and acceleration distances compatible with half of the shuttle run overall distance. The dashed vertical lines delimit the zone of interest, here, for (half of) the shuttle run of 20+20m, down to 5+5m (see text for further details). The thick white curve refers to the apparent efficiency of positive muscle work according to the measured mechanics of gradient running (Minetti et al., 1994), time confined to the zone of interest. The inset graph at the top-left of the figure shows reconstructed first leg of the shuttle runs of different distances, with the accelerative phase followed by the decelerative one. The output of the program includes the average values of metabolic cost due to speed changing and of the average apparent efficiency of positive work of the entire shuttle run.

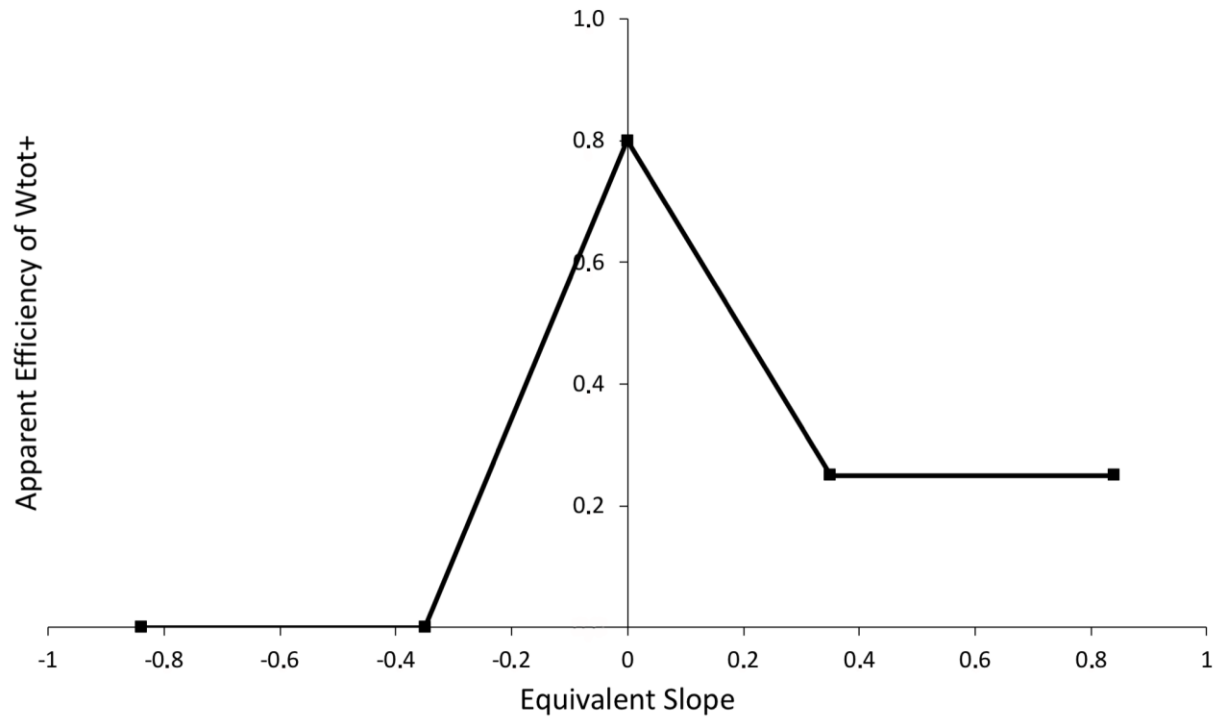


Figure 3: The apparent efficiency of positive muscle work vs ES. This graph is the result of investigations about running mechanics on level (Cavagna and Kaneko, 1977), on positive and negative gradients (Minetti et al., 1994) and of the proposed rationale to link it to accelerations and decelerations found in SR (see text).

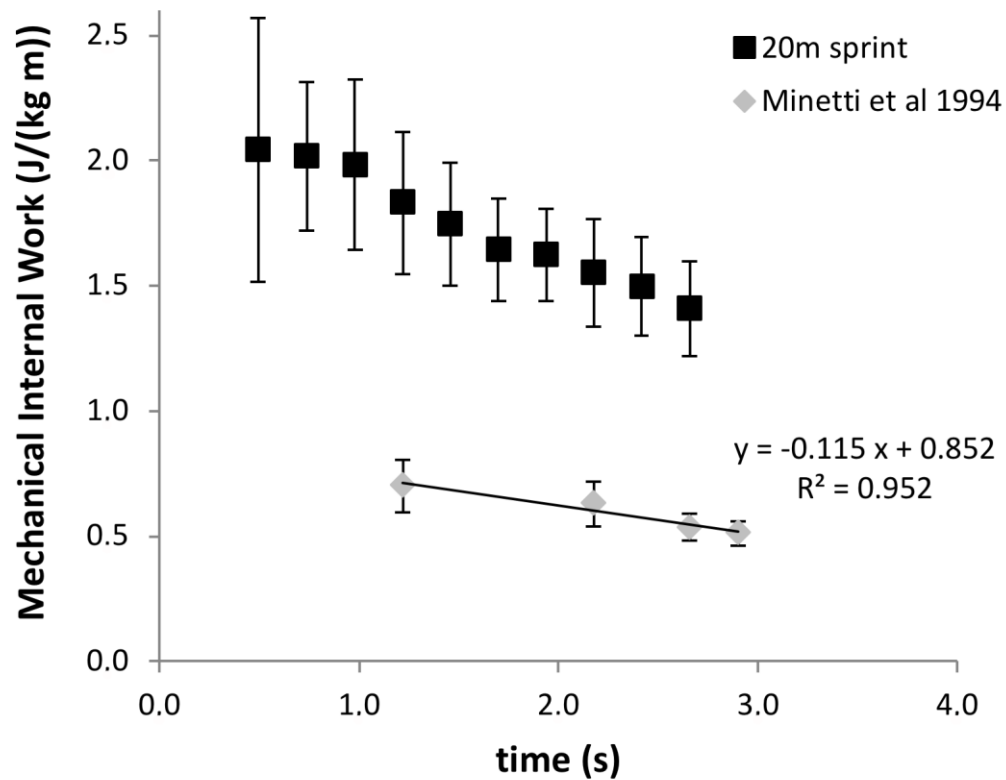


Figure 4: Step by step mechanical Internal Work (\pm SD, black squares) during the acceleration phase of a maximal 20m sprint (Pavei et al., 2018), and in treadmill running at different uphill slopes at constant submaximal speed (\pm SD, grey diamonds, Minetti et al., 1994) are compared. Each point of the second data series has been time-aligned in the graph as to appear where a step of the sprint sequence, because of its instantaneous a_f , reports an ES very close to the uphill treadmill slope.

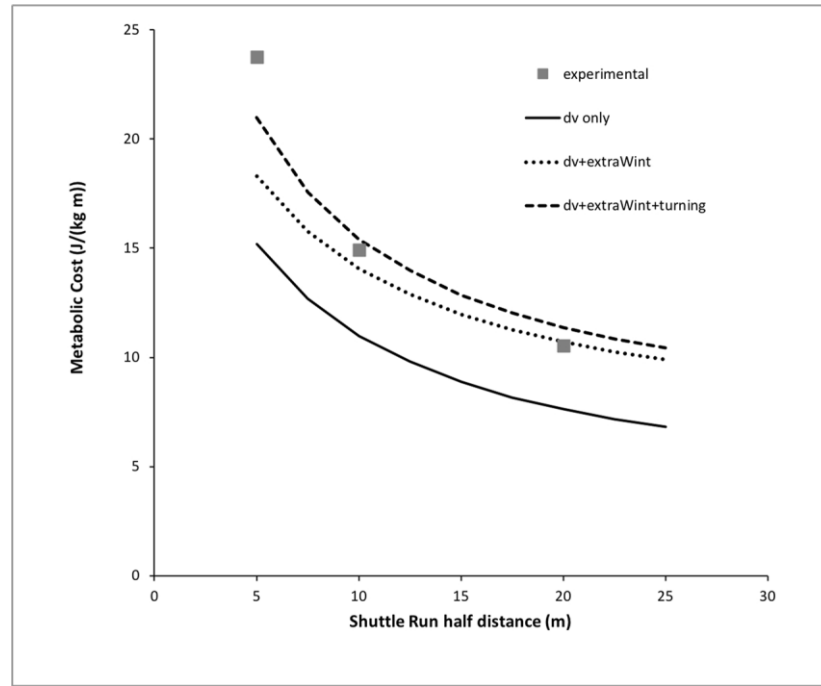


Figure 5: Experimental (close squares) and estimated metabolic cost of SR as a function of (half) distance: predictions are shown as just ES (dv only: continuous line), ES+extraWint (dotted curve), ES+extra Wint+turning cost (dashed line).

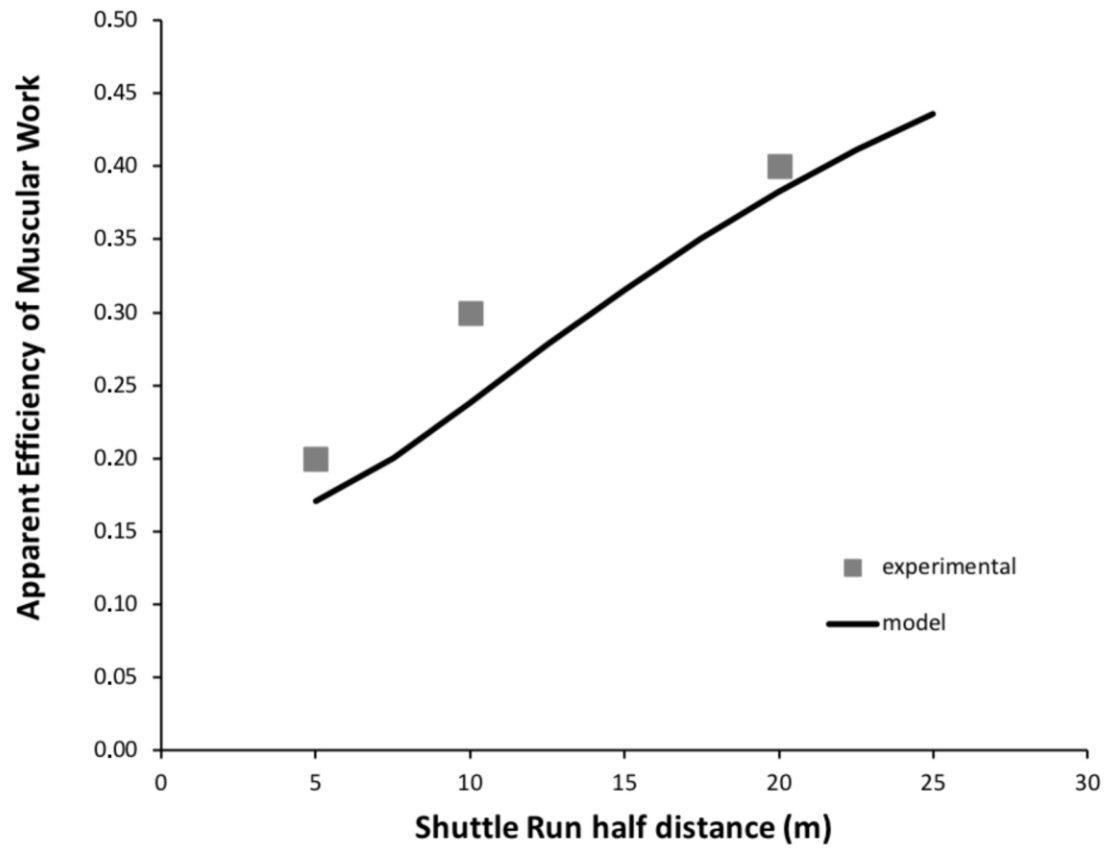


Figure 6: Experimental and estimated apparent efficiency (of muscle positive work) of SR as a function of (half) distance.