Modeling Observed Animal Performance Using the Weibull Distribution

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Key Words

Gecko, Adhesion, Lizard, Bite Force, Maximum Performance

Summary Statement

Using the Weibull Distribution to model observed animal performance data allows for more robust statistical analyses and estimates of maximum performance.

Abstract

Understanding how organisms adapt requires linking performance and microhabitat. However, measuring performance, especially maximum performance, can sometimes be difficult. Here we describe an improvement over previous techniques of only considering the largest observed values as maxima. Instead, we model expected performance observations via the Weibull distribution, a statistical approach that reduces the impact of rare observations. After calculating group-level weighted averages and variances by treating individuals separately to reduce pseudoreplication, our approach resulted in high statistical power despite small sample sizes. We fit lizard adhesive performance and bite force data to the Weibull distribution and found it to closely estimate maximum performance in both cases, illustrating the generality of our approach. Using the Weibull distribution to estimate observed performance greatly improves upon previous techniques by facilitating power analyses and error estimations around robustly estimated maximum values.

<u>Introduction</u>

Studies of ecological morphology and evolution often link an organism's morphology, performance, and ecology to suggest adaptation (Wainwright and Reilly, 1994). These studies typically assume accurate measurements of performance, which can be difficult to obtain, sometimes with a few observations contributing to a final point estimate of performance, especially when maximum performance values are of interest (Anderson et al., 2008; Garland and Losos, 1994). For the initial approval of projects by animal use and collection review boards, it is also necessary to plan for the number of individuals to be tested, the testing procedure for each individual, and the resulting statistical power of the results. Power analyses can describe the number of trials and individuals needed, but require an explicit statistical distribution of results, which is difficult to estimate when quantifying maximum performance. Here we describe a modeling approach using the Weibull distribution. In this approach, maximum performance is estimated as a parameter of the Weibull distribution. This approach is more robust than current methods of quantifying performance, which consider only a subset of maximum observations. The Weibull approach minimizes the effect of rare events and allows the use of additional statistical tools.

The Weibull distribution is valid for data that are likely to be non-normal, such as estimates of a maximum value, which would be expected to produce a skewed distribution of observations. While the Weibull distribution has been used occasionally to model behavior (Britten et al., 1992; Davis, 1996; Pugno and Lepore, 2008; Simpson and Ludlow, 1986), it is most often used in material science to predict mechanical failure. In this scenario, the Weibull distribution is modeling the likelihood of an observed event (i.e. mechanical failure) as it relates to some other factor (i.e. applied force or time), but this does not need to be the case. The Weibull distribution is highly flexible and can model many different patterns (Cornwell and Weedon, 2014; McCool, 2012; Weibull, 1951; Yang and Xie, 2003). This differs from the exponential distribution, which assumes a constant event rate and the normal distribution, which can only model non-skewed data. The Weibull distribution includes a scale parameter (λ) and a shape parameter (k, also known as the Weibull

modulus). The scale parameter (λ) has the same units as the modeled data and dictates the distribution's location on the x-axis. λ is closely related to the distribution's mode when $k > 1$ (Fig. 1). The dimensionless shape parameter (k) loosely controls the width and shape of the distribution for a given λ (Fig. 1). With these parameters, the Weibull distribution can model left and right skewed distributions (when k > 1) as well as exponential distributions (when k ≤ 1). Here we use the λ parameter as our performance value estimate.

To investigate the use of the Weibull distribution, we quantified the adhesive performance of gecko lizards by measuring toe detachment angle (Fig. 2; Autumn et al., 2006; Hagey et al., 2014). We first compared the fit of our observations to multiple distributions. We then considered how to model multiple observations from multiple individuals to avoid pseudoreplication and produce group-level estimates. We also conducted a power analysis to identify the minimum number of trials per individual and number of individuals needed to detect differences between groups. Lastly, we explored the generality of our approach by using the Weibull distribution to analyze previously published lizard bite force data.

Materials and Methods

To evaluate the use of the Weibull distribution, we used two empirical toe detachment datasets from the lizard *Gekko gecko*. We used a multi-individual dataset that included 206 observations from 13 individuals with an overall average of 15.4 trials per individual (max: 40 trials per individual, min: four trials per individual). Our second dataset, a subset of the first, was comprised of 40 observations from a single individual. To quantify toe detachment angle, we suspended live lizards from a single rear center toe (Schulte et al., 2004; Wang et al., 2010; Zani, 2000) from a slowly inverting glass microscope slide, recording the surface angle at which the lizard spontaneously detaches (Fig. 2; Autumn et al., 2006; Emerson, 1991; Hagey et al., 2014). As the glass surface inverts, the probability of detachment increases, making angle of toe detachment an excellent assay to be modeled by the Weibull distribution.

We preliminarily compared the fit of the normal, Weibull, exponential, gamma, and log-normal distributions qualitatively using Q-Q plots (Fig. 3). Previous studies have shown that the Weibull distribution can sometimes be difficult to distinguish from other distributions (Bain and Engelhardt, 1980; Fearn and Nebenzahl, 1991). We also compared the fit of these distributions to our data employing a bootstrapped Kolmogorov–Smirnov test with 5000 bootstraps to alleviate issues with repeated values (Table 1; Sekhon, 2011). The K-S test evaluates the probably that our observed values could have been drawn from each considered distribution. We also compared Akaike Information Criterion weights with a correction for small sample size (AICc; Burnham and Anderson, 2002).

To evaluate multiple observations from multiple individual lizards, we needed to determine how variable our observed individuals were and if they could be modeled together under one distribution. We fit the Weibull distribution to our multi-individual dataset (our null model, all observations lumped together) or allowed each individual lizard to have its own set of parameters (our alternative model). We then conducted a likelihood ratio test and used the chi-squared distribution to determine if we should reject our null model. Lastly, we fit a parametric survival regression model using the Weibull distribution with individual as a factor, predicting detachment angle using the R library *survival* (Therneau and Grambsch, 2000).

We then calculated a weighted group average detachment angle ($\bar{x}$) using Eq. 1, where n is the number of individuals, λ_i is individual i's estimated scale parameter value, and w_i is individual i's weighting:

$$\bar{x} = \frac{\sum\limits_{i=1}^{n}(\lambda_i w_i)}{\sum\limits_{i=1}^{n} w_i}. \qquad\qquad \text{Eq. 1}$$

We calculated each individual's weightings (w_i) using the estimated error, σ_i^2, around their scale parameter value λ, via the equation:

$$w_i = \frac{1}{\sigma_i^2}. \qquad\qquad \text{Eq. 2}$$

To calculate the variance around our weighted mean, $\sigma_{\bar{x}}^2$, incorporating within and between individual variation, we used the following equation (Bevington and Robinson, 2003):

$$\sigma_{\bar{x}}^2 = \frac{1}{\sum_{i=1}^{n}\left(\frac{1}{\sigma_i^2}\right)} \times \frac{1}{n-1} \times \sum_{i=1}^{n}\frac{\left(x_i - \bar{x}\right)^2}{\sigma_i^2}. \qquad \text{Eq. 3}$$

It is worth noting that when the individual errors (σ_i^2) are all equal, Eq. 3 simplifies to the standard error around an unweighted mean.

Using the above-described analysis approach, we simulated performance data and conducted power analyses to investigate the effect of sample size on our ability to detect differences in detachment angle between groups (Fig. 4). In order to simulate the necessary data, we first needed to obtain realistic k and λ values. Due to the Weibull distribution's heteroscedastic nature, k is affected by changes to λ (Fig. 1; McCool, 2012). In addition, k is sensitive to sample size, with smaller datasets fitting distributions with larger k values i.e. narrower distributions. To confirm the correlated relationship between k, λ, and sample size, we used a previously collected dataset of toe detachment observations from 53 gecko and anole species (244 individuals with an mean of 9.0 trials per individual). This dataset was collected using similar methods as described above including measurements from the lab and field using captive and wild caught specimens (Autumn et al., 2006; Hagey et al., 2014). We estimated k and λ for each individual and fit these data to a linear model where k was predicted by λ, number of trials, and an interaction between λ and the number of trials. We found both λ (p < 0.0001) and number of trials (p < 0.0001) to significantly predict k. The interaction term was not significant and removed from the analysis. The coefficients from this linear model were then used to estimate realistic k values for a given λ and number of trials (n):

$$k = 0.6\lambda - 0.53n + 10.2. \qquad \text{Eq.4}$$

We then produced multiple datasets of simulated toe detachment data using the Weibull distribution. Each dataset was composed of simulated data from two groups with different assigned λ parameter values to see if we could significantly

differentiate small differences between groups The differences between groups were one, two, three, or five degrees (i.e. our effect sizes). We chose to compare groups with λ values of 15° versus 15° plus an effect size and 25° versus 25° plus an effect size based on empirical observations. We also evaluated comparisons in which groups had 20 trials per group distributed across one individual, two individuals, and four individuals or 50 trials per group distributed across two individuals, five individuals, and ten individuals (Fig. 4). To statistically compare our estimated weighted averages between simulated groups, we tested whether either of the groups' means were within 1.96 standard deviations (square root of the estimated mean variance multiplied by 1.96) of the other. We then calculated the percentage of our 1000 replicate trials that produced significantly different comparisons (Figure 4). All analyses were conducted using the R Studio statistical software (Version 0.98.501, 2013 RStudio, Inc.).

Results and Discussion

We qualitatively compared our data to the normal, Weibull, exponential, gamma, and log-normal distributions (Fig. 3). Our Q-Q plots strongly suggest that our data differ from what would be expected under the exponential distribution because high angle observations were rare. The fitted normal, gamma, and log-normal distributions all yielded similar results, suggesting, low-angle observations were more common than if our data were drawn from these distributions, i.e. our data are left-skewed, which is to be expected when attempting to observe a maximum value. The Q-Q plots suggest that the Weibull distribution closely approximates our observed data. An important distinction between the Weibull and normal distributions is that the Weibull predicts rare early failure events, producing a negatively skewed distribution of observations, while the normal distribution does not. We also evaluated the normal, exponential, Weibull, gamma, and log-normal distributions' goodness of fit using a bootstrap version of the Kolmogorov-Smirnov test (Sekhon, 2011). Our multi-individual dataset was significantly different than all the distributions considered, while our individual dataset was not significantly different from any of the distributions considered, except the exponential distribution (Table 1). These results may be due to the fact that our multi-individual dataset represents a collection of observations from multiple individuals, each with their own performance distribution (see below). Conversely, our individual dataset may be too small to distinguish between distributions (see Methods). Akaike Information Criterion weights, corrected for small sample sizes (Burnham and Anderson, 2002) provided more clear results, suggesting the Weibull distribution is the best fitting distribution considered with an AICc weight of 0.94 and 0.81 for our multi-individual and individual datasets respectively (Table 1).

To estimate a group-level detachment angle, we needed to consider multiple observations from multiple individuals, while also limiting pseudoreplication. We found significant support to reject our null hypothesis (lumping all our observations together, D = 100.3, df = 24, p < 0.0001). We also fit a parametric survival regression model using the Weibull distribution in which individual, as a factor, significantly predicted detachment angle (p < 0.0001), again suggesting it is better to model

performance observations of each individual lizard separately (Therneau and Grambsch, 2000). By fitting each individual to a distribution and using each individual's parameter values and error to estimate weighted group averages, we prevent pseudoreplication and reduce the impact of unbalanced sampling, penalizing individuals with large error estimates due to small sample sizes or erratic observations.

We also investigated the trade-off between the number of trials per individual and the number of individuals tested using power analyses. We found abundant power to detect small differences between groups (effect sizes of one to two degrees) even with relatively small datasets, i.e., few total trials, few individuals, or few trials per individual (Fig. 4). In our simulations using 50 total observations, we observed an overall increase in power, as to be expected with more data regardless of partitioning. Fitting the Weibull distribution to datasets with five or fewer observations per individual, especially if there is little variation among the observations, can hinder the estimation of parameter estimates. Although our power analyses suggested we could detect group differences with few trials or individuals, we recommend six to ten trials per individual to assure a successful fit of the Weibull distribution. Using data from two individual lizards with ten trials each, we consistently had enough power to detect a difference of one degree 80% of the time, although we recommend sampling more individuals to better capture inter-individual variation.

In addition, we did not observe differences in power between datasets considering low detachment angles (15°) and high detachment angles (25°). We believe this is due to the fact that we simulated our data using values of k that complemented λ and the number of trials per individual (see Methods). We conducted similar analyses with a constant k value regardless of λ and number of trials per individual and found power decreased when considering higher values of λ. This is because at higher λ values, with a constant k, the Weibull distribution has a larger variance (Fig. 1), reducing the power to detect differences between groups.

To investigate the generality of the Weibull distribution's ability to model animal performance, we also compiled published bite force data from *Anolis*

carolinensis (provided by A. Herrel; Irschick et al., 2005a; Irschick et al., 2005b). This dataset was comprised of 381 individuals, each with 5 trials each. We found each individual's estimated λ values to be similar to the mean of the best three trials with the added benefit of being robust to extreme values (Fig. 5).

In conclusion, using the Weibull distribution as an expected distribution of observed performance is a more robust approach compared to only using a subset of trials. We strongly encourage researchers to investigate the fit of the Weibull distribution to their performance datasets. With the addition of estimating weighted group means, comparisons between treatment and control groups or comparisons between species are easily facilitated. If making comparisons across species, once species means have been estimated, phylogenetic non-independence can then easily be taken into account.
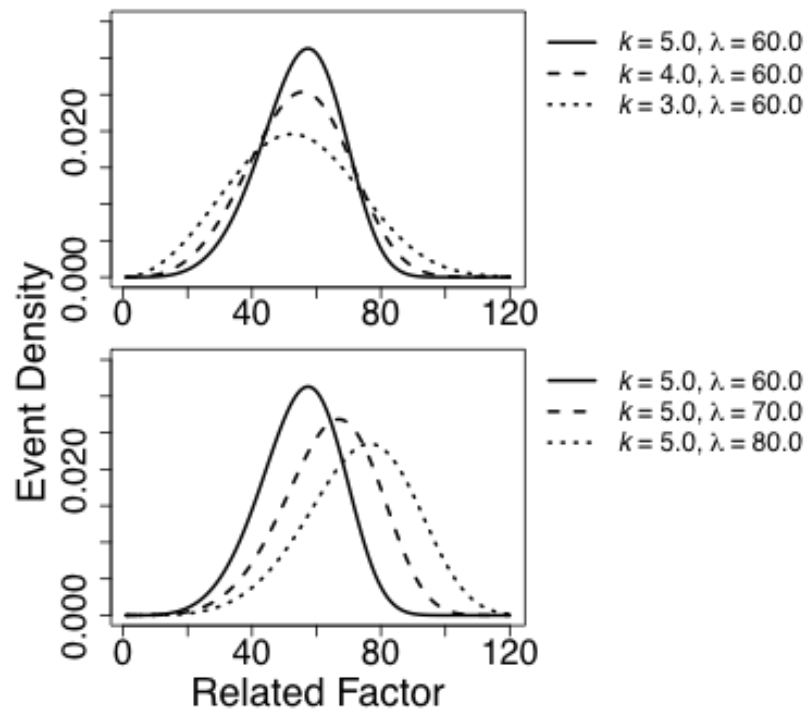
Figures

Figure 1. Weibull Distribution and Parameter Variation

The shape (k) and scale (λ) parameters of the Weibull distribution interact to dictate the shape and location of the distribution. Here we display relevant parameter combinations illustrating the relationship between positive k values and λ.
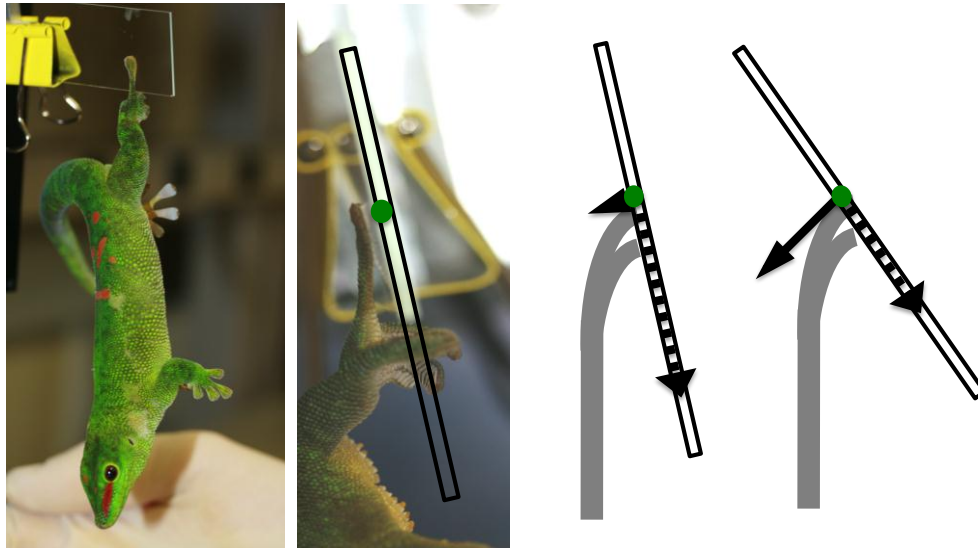
Figure 2. Angle of Toe Detachment Assay

We used toe detachment angle to quantify the adhesive performance of padded lizards. In this assay, a lizard is suspended from a glass surface by a single rear toe via their natural adhesive properties (left images). When the surface is near vertical, the lizard is generating predominantly friction relative to the substrate (see right images, adhesive microstructures illustrated in gray, friction illustrated as dashed arrows, adhesion illustrated as solid arrows). As the substrate is slowly inverted, less friction and more adhesion relative to the substrate is generated (see far right image). At the angle of toe detachment, the adhesive microstructures can no longer maintain the proper orientation with the surface and the animal falls onto a cushioned base. As a result, the angle of toe detachment quantifies the maximum amount of adhesion, relative to friction; an individual is capable of generating (Image modified from Hagey et al., 2014).
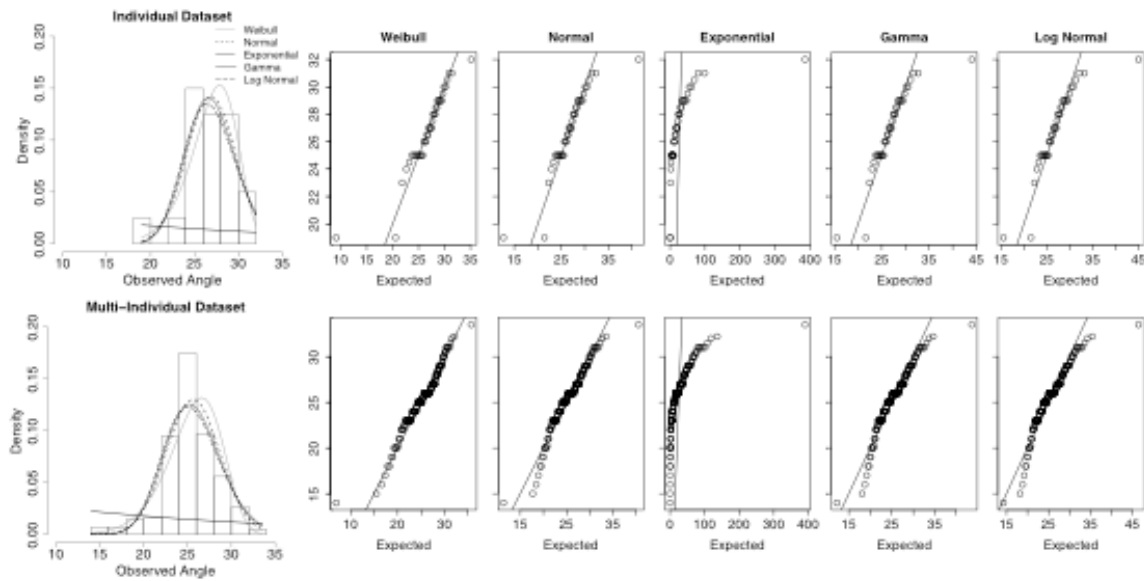
Figure 3. Fit of Empirical Data to the Weibull

To determine the qualitative fit of our empirical data to the Weibull distribution, we generated Q-Q plots of our two toe detachment datasets. The top plots use observations from a single individual with 40 trials. The lower plots consider our larger dataset containing 206 trials from 13 individuals. The Q-Q plots also contain a line at y = x as reference. The histograms on the left display our data with fitted distributions. The Weibull, normal, exponential, gamma, and log-normal distributions are displayed as solid grey, dashed black, solid black, dotted black, and dash dotted black respectively.
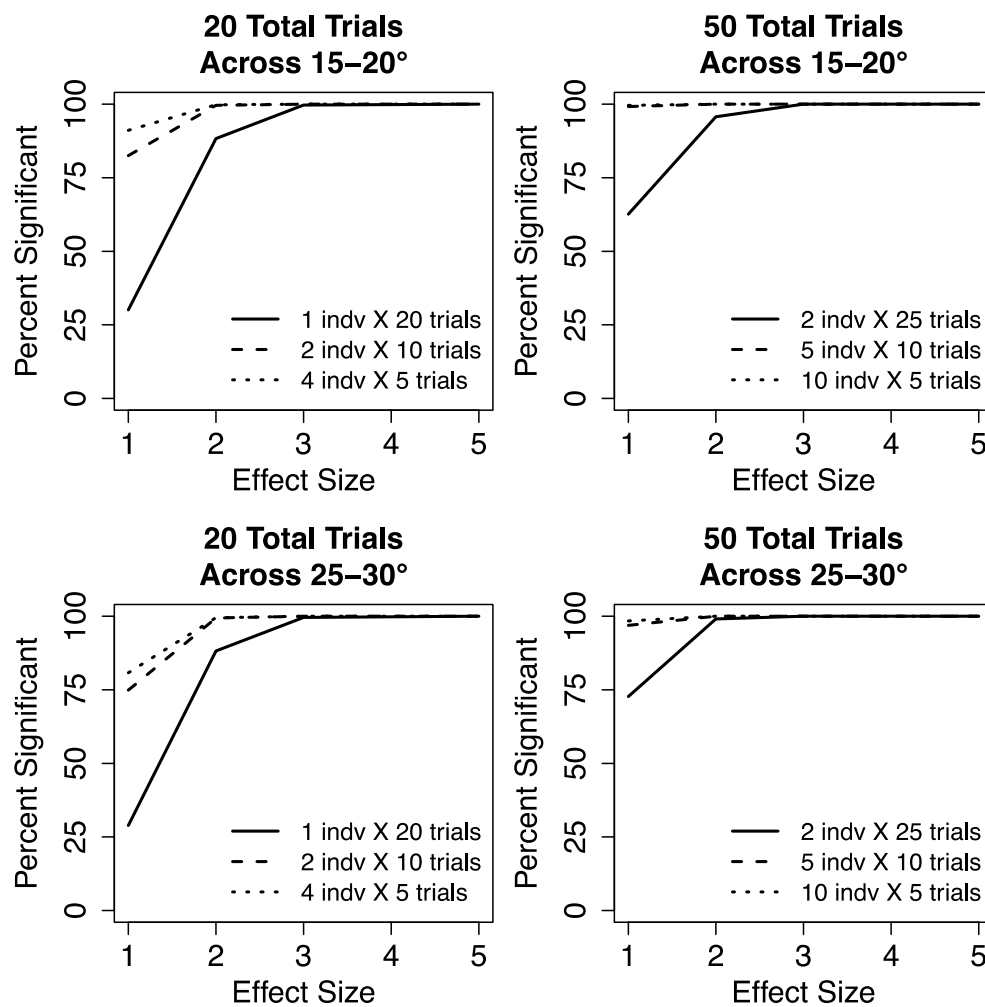
Figure 4. Power Analyses

We considered groups with low detachment angles (15° plus effect size, upper plots), and high detachment angles (25° plus effect size, lower plots). We also considered datasets using 20 total trials (left plots) and 50 total trials (right plots). The y-axes represent the percentage of our 1000 replications that were significantly different.
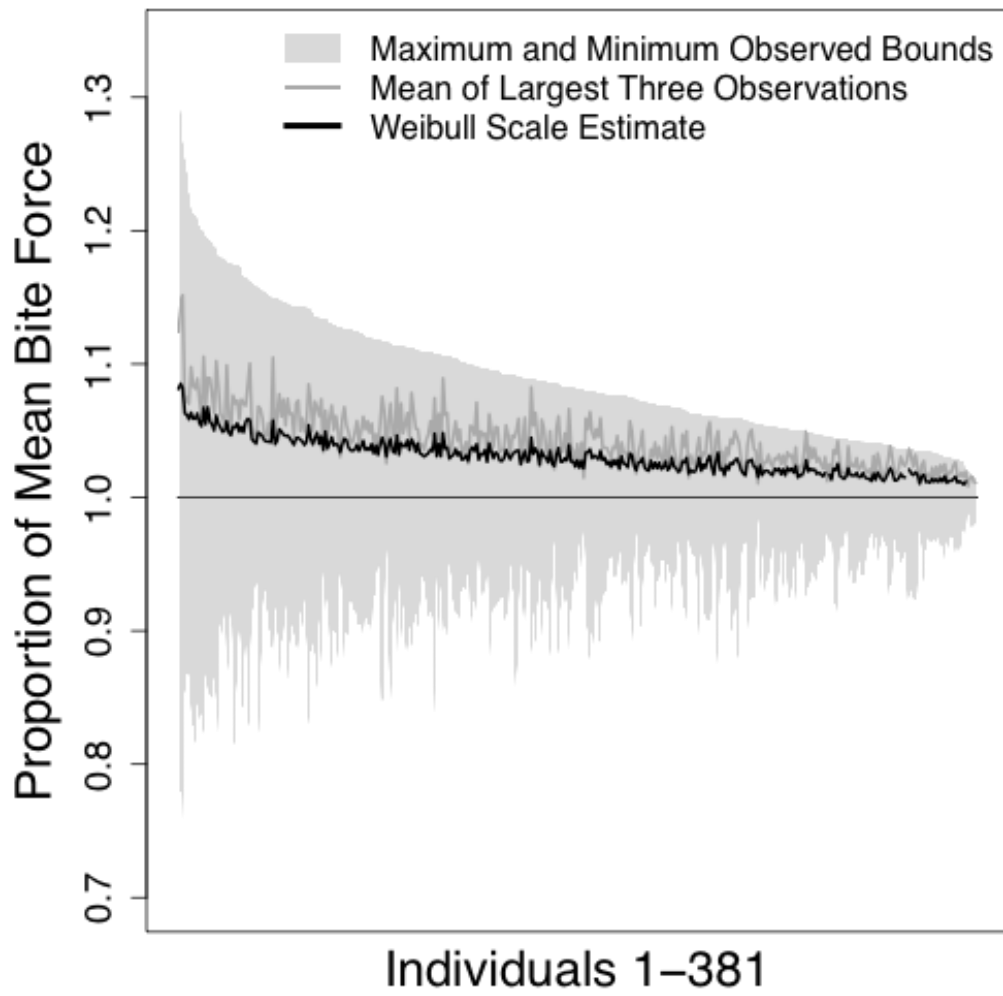
Figure. 5. Lizard Bite Forces

Using a compiled dataset of published lizard bite forces, we estimated each individual Weibull scale (λ) parameter (black points) and plotted these values with each individuals' observed maximum and minimum observation (shaded area) and the mean of the three largest observations (grey points). All calculated values were divided by each individual's mean observation to similarly scale the dataset.

Table

Table 1. Toe Detachment Angle and Model Selection

We evaluated multiple distributions to determine which best modeled our observed toe detachment angles using K-S tests and AICc weights. All of the model distributions evaluated had two free parameters, except the exponential, which had one.

Individual Dataset

Distribution	K-S Test p-value	Log Likelihood	AICc	AICc Weight
Weibull	0.733	-96.3	196.9	0.81
Normal	0.461	-98.1	200.5	0.14
Exponential	0.000	-171.7	345.5	0.00
Gamma	0.443	-99.5	203.2	0.03
Log-Normal	0.378	-100.3	204.8	0.02

Multi-Individual Dataset

Distribution	K-S Test p-value	Log Likelihood	AICc	AICc Weight
Weibull	0.008	-519.8	1043.7	0.94
Normal	0.009	-522.6	1049.2	0.06
Exponential	0.000	-874.1	1750.1	0.00
Gamma	0.001	-530.5	1065.1	0.00
Log-Normal	0.000	-535.8	1075.7	0.00

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