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Redirection of center-of-mass velocity during the step-to-step transition of human walking

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SUMMARY

Simple dynamic walking models based on the inverted pendulum predict that the human body's center of mass (COM) moves along an arc during each step, with substantial work performed to redirect the COM velocity in the step-to-step transition between arcs. But humans do not keep the stance leg perfectly straight and need not redirect their COM velocity precisely as predicted. We therefore tested a pendulum-based model against a wide range of human walking data. We examined COM velocity and work data from normal human subjects (N=10) walking at 24 combinations of speed (0.75 to 2.0 $m\,s^{-1}$) and step length. These were compared against model predictions for the angular redirection of COM velocity and the work performed on the COM during redirection. We found that the COM is redirected through angular changes increasing approximately linearly with step length (R^2=0.68), with COM work increasing with the squared product of walking speed and step length (R^2=0.82), roughly in accordance with a simple dynamic walking model. This model cannot, however, predict the duration of COM redirection, which we quantified with two empirical measures, one based on angular COM redirection and the other on work. Both indicate that the step-to-step transition begins before and ends after double support and lasts about twice as long – approximately 20–27% of a stride. Although a rigid leg model can predict trends in COM velocity and work, the non-rigid human leg performs the step-to-step transition over a duration considerably exceeding that of double support.

Key words: locomotion, biomechanics, center of mass, velocity, redirection, step-to-step transition, walking, work, inverted pendulum, leg compliance, hodograph, dynamic walking, step length.

INTRODUCTION

The body center of mass (COM) moves like an inverted pendulum during human walking. A pendulum conserves mechanical energy and needs no work to move along an arc (Alexander, 1991). By keeping the knee relatively straight, the human stance leg also supports body weight with relatively little muscle force (Kuo, 2007). These mechanical savings appear to be physiologically relevant because work production and force development in muscle both require metabolic energy expenditure. The pendulum offers strong insight regarding the mechanism of walking, but it also has limitations. The human stance leg is not perfectly rigid, and the actual COM motion deviates somewhat from a pendulum's arc. Here we examine the pendulum model's predictions for how the COM is redirected between arcs, to quantify how well it predicts human COM velocity and to determine its limitations as a predictive model of human walking.

The pendulum analogy is demonstrated well by fluctuations of COM energy, as the kinetic energy fluctuates out of phase with gravitational potential energy during walking (Cavagna and Kaneko, 1977). By contrast, these fluctuations occur in phase during running (Cavagna et al., 1977), where a spring-mass analogy better describes the compression and extension of the stance leg (Blickhan, 1989). The phasing and amplitude of energy fluctuation also change during walking, as a function of gait parameters such as speed, step length and step frequency (Willems et al., 1995). Some of these changes may be attributed to work performed on the swing leg, but some may also be associated with the stance leg when it does not behave as a perfectly rigid pendulum. This makes it difficult to predict how gait parameters will affect energy fluctuations and, conversely, to relate observed fluctuations to actual COM motion.

The imperfect rigidity of the stance leg has previously been noted in several ways. Alexander pointed out that the ground reaction forces under each leg are explained much better by a model with axially compliant legs (Alexander, 1992). Humans produce a characteristic vertical ground reaction force profile with two peaks that are not produced by a rigid inverted pendulum model. The addition of compliance, similar to the spring-mass running analogy, can predict such a profile in forward dynamics simulations (Geyer et al., 2006). These findings are corroborated by inverse dynamics analyses, which show that a 'telescoping pendulum' (referring to axial lengthening and shortening) allows for much better matching of ground reaction forces than a rigid one (Buczek et al., 2006). The leg joints, most notably the knee, have long been observed to flex and extend during single support (Winter, 1991), and the telescoping action is a simple means of summarizing the effect of multiple joint motions on COM motion. None of these effects, however, are easily quantified through energy fluctuations.

Another limitation of the inverted pendulum analogy is that it only applies to the single support phase of walking. Double support is not pendular and instead functions as a transition between single support phases (Donelan et al., 2002a). The step-to-step transition redirects the COM velocity from the downward portion of an arc prescribed by the stance leg for one step, to the upward portion of another arc prescribed by the succeeding stance leg for the next step. Assuming rigid stance legs and impulsive collisions, simple models of dynamic walking predict that these velocity changes determine the mechanical work performed during the transition

(Adamczyk et al., 2006; Donelan et al., 2002b; Kuo, 2002). Empirical data suggest that this work exacts an approximately proportional metabolic cost in humans (Donelan et al., 2001; Donelan et al., 2002a). These models do not, however, capture how human legs are imperfectly rigid and unable to produce ideal, instantaneous impulses. This can potentially lead to incorrect predictions depending on the degree and nature of axial leg motion. Previous studies have focused on the work of the step-to-step transition – primarily during double support – but have largely overlooked the possible dependency of COM velocity changes on axial leg motion during single support.

The purpose of this study was to examine how COM velocities vary as a function of gait parameters such as speed, step length and step frequency. We measured COM velocities between pendulum-like phases, across a wide range of walking gait parameters. We analyzed the relationships between velocity magnitudes and directions, the impulses provided by the two legs, and the mechanical work performed on the COM during the transition between steps. These were then compared against the predictions of simple models assuming rigid legs. It is possible that human legs deviate from rigid leg models, and the consistency of that deviation may determine how useful simple models are for predicting general trends in COM velocity redirection and the associated work.

MATERIALS AND METHODS

We compared experimental measurements of COM motion during human walking against simple mathematical predictions. Experimental measurements were made for a range of slow to fast walking speeds, short to long steps, and low to high step frequencies, the extremes of which might be most expected to induce stance leg behavior not resembling a pendulum. Predicted quantities included the magnitude and direction of COM velocity, the impulse produced by each leg against the COM, and the work performed on the COM through each leg. The predictions were based on a simple model of dynamic walking that relies entirely on passive dynamics except for active push-off to produce gait. This model predicts trends in the measured quantities as a function of walking speed and step length. This section begins with a brief summary of the model, followed by descriptions of the experimental conditions, associated measurements and quantitative analyses.

Model

We used a previously developed dynamic walking model (Kuo, 2002) to derive the dependency of COM motion on gait parameters. The model is a simplification of human gait that divides the COM's approximately sinusoidal motion into two portions: an upper one that corresponds roughly with single support and a lower one for double support. The upper portion is that which most resembles a pendulum's arc (see Fig. 1A), with the COM velocity directed upward at the beginning and downward at the end. The forward speed reaches minimum at the top of the arc, when the pendulum has maximum gravitational potential energy. We treat the lower portion of the path as a redirection phase (Fig. 1B) or step-to-step transition (Donelan et al., 2002b) that ends with the beginning of the next pendulum phase. The model assumes perfectly rigid legs of negligible mass supporting body mass concentrated at the pelvis, so that the COM moves atop a simple inverted pendulum (see Fig. 1C). Each foot has sufficient mass to allow the swing leg to behave like a pendulum, but with negligible effect on the rest of the body. The model's step-to-step transition (Fig. 1D) consists of push-off and collision, treated as successive, instantaneous impulses applied along the trailing and leading leg, respectively. These impulses perform all of the work in the model, determined entirely by the COM velocities at beginning and end of the pendulum phase. Humans do not produce such forces, but the work performed by the model obeys similar trends for more realistic forces, as long as the actual duration and displacement of the step-to-step transition are relatively small.

The principal predictions are for fluctuations in COM velocity and work performed on the COM by the individual legs. These come as a series of linear relationships, all of which may be described intuitively with the pendulum model. The model constrains the COM trajectory along a series of pendular arcs, each with an angular excursion determined by step length (Fig. 2A). This angular excursion also dictates the directional change, δ, that the COM velocity must undergo in the step-to-step transition. Walking faster at a given step length (by increasing step frequency) produces a higher velocity along the same trajectory. The magnitude of COM velocity increases approximately linearly with the model's walking speed (Fig. 2B), for any step length. The angular redirection δ increases approximately linearly with the model's step length, for any walking speed. The work performed on the COM during the step-to-step transition is proportional to the change in kinetic energy due to the push-off and collision. This energy change is, in turn, proportional to the square of COM velocity change and therefore to the squared products of COM velocity magnitude and the angular redirection. Combining all of these relationships, the COM work per step is predicted to be proportional to the squared product of walking speed and step length.

The mathematical details of the work prediction are as follows. We refer to average walking speed as $\bar{v}$, and the COM velocities at the beginning and end of the step-to-step transition as v_{pre} and v_{post}, respectively (see Fig. 1D; Fig. 2A). Model simulations (Kuo, 2002) show that:

$$v_{\mathrm{pre}} \propto \bar{v}\,, \tag{1}$$

and that best economy is achieved if the trailing leg applies a push-off impulse, sufficient to reduce the vertical component of COM velocity to zero, immediately before heel-strike. The heel-strike collision then produces an impulse along the leading leg, such that v_{post} is directed along a new arc-like trajectory prescribed by that leg. The angle δ between the legs increases with step length, s:

$$\delta \propto s\,, \tag{2}$$

for small angles. The work, W, performed by such a push-off (and the negative work performed by the collision) is equal to the change in kinetic energies before and after push-off:

$$W = \tfrac{1}{2}M\,(v_{\mathrm{mid}}^2 - v_{\mathrm{pre}}^2) = \tfrac{1}{2}Mv_{\mathrm{pre}}^2 \tan^2 \delta\,, \tag{3}$$

where v_{mid} refers to the mid-transition COM velocity between the push-off and heel-strike impulses, and M refers to body mass. Again, assuming small angles:

$$W \propto (v_{\mathrm{pre}} \cdot \delta)^2\,. \tag{4}$$

Combining Eqns 1, 2 and 4 yields work per step in terms of speed and step length:

$$W \propto (\bar{v} \cdot s)^2\,. \tag{5}$$

Although these relationships are derived with the very simple model presented here, the addition of human-like mass distribution and arc-shaped feet has previously been shown to have little effect on the overall linear form of the predictions except for an added constant offset term to the proportionalities (Adamczyk et al., 2006; Kuo, 2001).

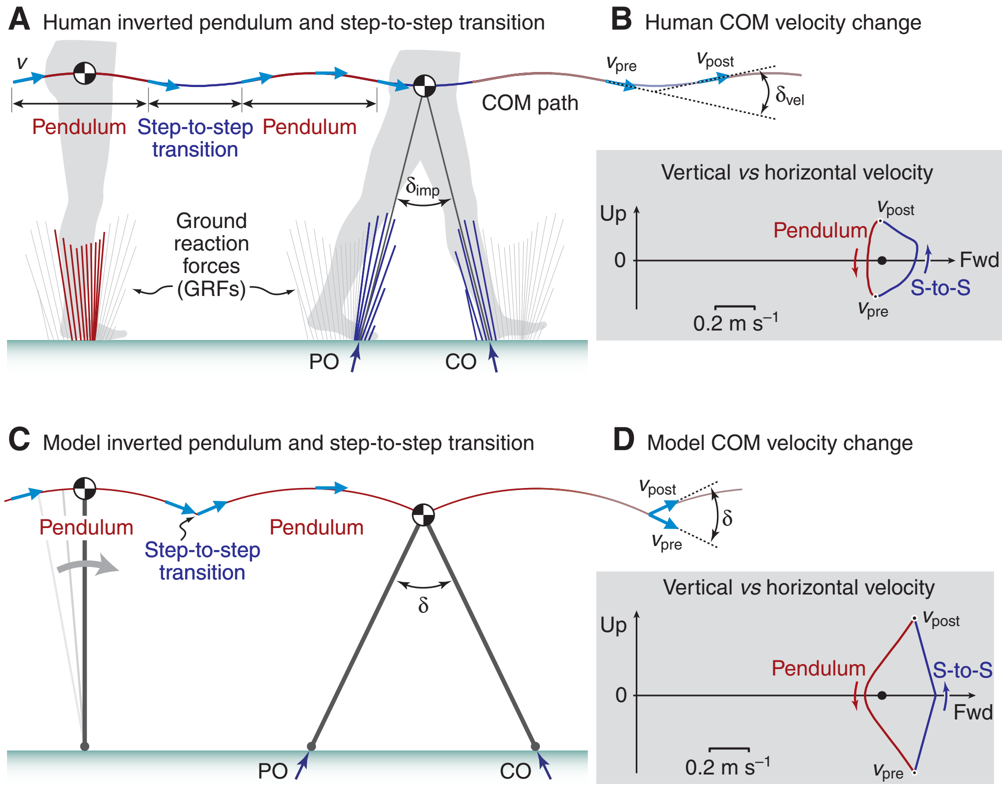

Fig. 1. Body center of mass (COM) motion in humans and in a simple walking model. (A) The human COM moves in a roughly sinusoidal path, which may be divided into inverted pendulum and step-to-step transition phases, corresponding approximately to single and double support, respectively. Ground reaction forces (GRFs) from each leg produce push-off and collision forces (highlighted with thicker lines during double support) that sum to the impulse vectors (labeled PO for push-off and CO for collision) separated by an angle, δ_{imp}. (B) The COM velocity changes during the step-to-step transition, from a pre-transition velocity (v_{pre}) to a post-transition velocity (v_{post}), separated by an angle, δ_{vel}. These changes may also be observed in a plot of vertical *vs* horizontal components of COM velocity (inset), which traces a counter-clockwise path, as the COM velocity changes from upward to downward during the pendulum phase, and then from downward to upward due to push-off and collision forces during the step-to-step transition (S-to-S). (C) In the model, COM velocity is prescribed by a simple pendulum, with an impulsive step-to-step transition. (D) The angular difference δ between the model's impulses is equal to the angular difference between v_{pre} and v_{post}. The plot of COM velocities, termed the 'COM hodograph' (inset), also traces a counter-clockwise path.

Experiment

We tested model predictions with measurements of COM velocity and work from a wide range of human walking. We imposed 24 different combinations of walking speed and step length on 10 human subjects (five male, five female), with body mass, M, averaging 68.9±12.2 kg (mean ± s.d.) and leg length, L, averaging 0.93±0.05 m, and observed the impact of changes to these gait parameters on pre-transition COM velocity v_{pre}, COM redirection, collision work performed on the COM (see Eqns 1, 2, 4, and 5) and timing of redirection. We measured ground reaction forces (GRFs) while subjects walked over ground and used these to compute the COM trajectory and collision work, W, over the course of a step, defined as heel-strike to opposite heel-strike.

We used four different sets of conditions to map each subject's performance across a range of speeds and step lengths surrounding normal walking (see Fig. 3). Prior to these conditions, we evaluated each subject's preferred step frequency, f^*, and step length, s^*, at a designated nominal speed, $\bar{v}^*$, of 1.25 m s^{-1} (where $\bar{v}^*=s^*\cdot f^*$), with speed measured by photogates. The first set of conditions consisted of natural walking (circles in Fig. 3), in which subjects walked over ground at speeds of 0.75, 1.00, 1.25, 1.50, 1.75 and 2.00 m s^{-1} (0.6 to 1.6 × $\bar{v}^*$), all at their own preferred step length and frequency for each speed (see Donelan et al., 2002b). In the second set of conditions, subjects walked at the same speeds but with a constant step frequency f^* (constant frequency, CF; squares in Fig. 3) set by a metronome (Donelan et al., 2002a). Because speed equals step length multiplied by step frequency, this protocol resulted in step lengths ranging from 0.6 to 1.6 × s^*. The third set of conditions was complementary to the second; subjects maintained their preferred step length, s^*, across the same range of speeds by stepping to a metronome at frequencies from 0.6 to 1.6 × f^* (constant step length, CS; diamonds in Fig. 3). In the final set of conditions, subjects varied both step frequency and step length in inverse proportion to maintain the specified speed, $\bar{v}^*$, matching their step frequency to a metronome beat ranging from 0.70 to 1.30 × f^* (constant speed, CV; triangles in Fig. 3). All the data we analyzed were collected in conjunction with earlier studies (Donelan et al., 2002a; Donelan et al., 2002b), in which subjects completed three trials per condition, with conditions applied in random order across all four sets. All human subjects provided their informed consent, as approved by the University of California Institutional Review Board.

COM velocity and work were estimated from GRF data. COM velocity was determined by integrating three-dimensional GRF data (Cavagna, 1975; Donelan et al., 2002b), with integration constants for each step based on an assumption of periodic gait. Velocity and force data were then used to calculate the instantaneous rate of work performed by each leg on the COM, defined as the dot product of each leg's GRF against COM velocity (Donelan et al., 2002b). The

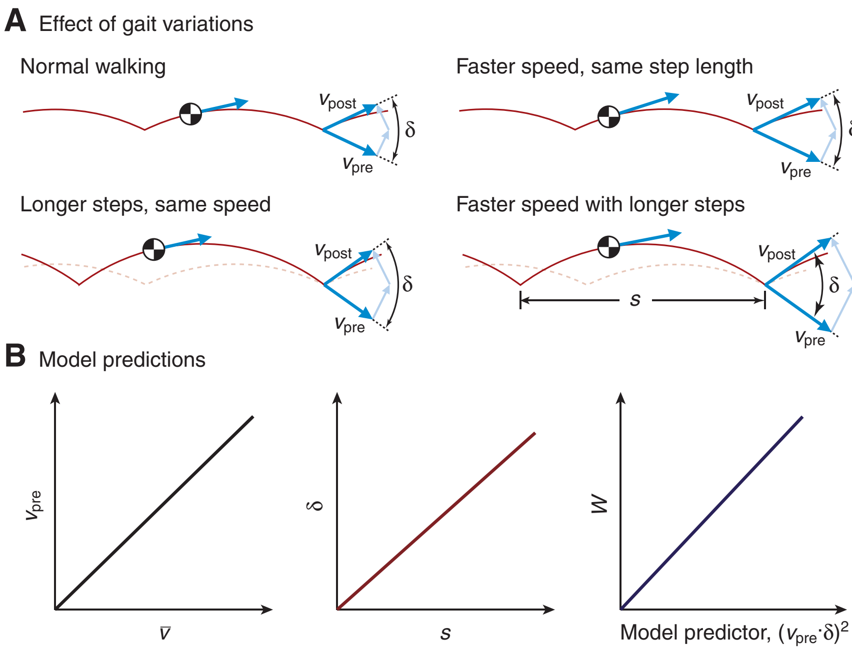

Fig. 2. (A) Effect of gait variations and (B) model predictions based on a simple dynamic walking model. (A) The magnitude of COM velocity increases with the model's walking speed, and its directional change, δ, increases with the model's step length, *s*. Faster walking speed and longer steps together require a greater change in COM velocity than either factor alone. (B) Model simulations predict that pre-transition velocity (v_{pre}) will increase approximately linearly with walking speed $\bar{v}$ (Eqn 1), angular redirection (δ) will increase approximately linearly with step length (*s*) (Eqn 2), and the associated work (*W*) performed on the COM will increase (Eqns 4 and 5) with a predicted quantity $(v_{pre}\cdot\delta)^2$, which is also proportional to the squared product of walking speed and step length $(\bar{v}\cdot s)^2$. The model predicts linear relationships with unknown slope and offset to be determined from experimental data.

work rate for each leg was then integrated to yield the positive and negative COM work performed by each leg during the step-to-step transition. To aid our analysis, we also plotted the vertical and horizontal components of COM velocity against each other over the course of a step (see Fig. 1B,D). The term 'hodograph' (Greenwood, 1988) refers to a plot of velocity components, and so we refer to our plot as a 'COM hodograph'.

A practical issue in the comparison of experimental data with model is that humans do not produce purely impulsive forces. The production of finite GRFs for a finite duration means that humans need not redirect their COM velocity by the same amount as the angle between the legs. We therefore defined separate quantities: δ_{vel} for the angular change in COM velocity, and δ_{imp} for the angular difference in ground reaction impulses. Both of these quantities are expected to increase with the model's δ, but not necessarily with equal proportions. Humans also need not produce equal amounts of work during push-off and collision. We therefore computed separate push-off positive work, W_{PO}, and (magnitude of) collision negative work, W_{CO}; we expected both to increase with the model's work, *W*.

Another issue is the duration of the human step-to-step transition. In the model, double support occurs in an instant, with push-off and collision impulses coinciding with redirection of the COM. In humans, double support occurs over a finite duration that only approximately matches when push-off and collision work are performed, which in turn only approximately coincides with the extremes of COM velocity redirection. We defined the duration of the step-to-step transition as the period, τ_{vel}, between extremes of direction for the COM velocity, referred to as v_{pre} and v_{post} (see Fig. 1B), locally surrounding double support (Adamczyk et al., 2006). This duration was then used to compute the velocity redirection and work measures. We also considered two additional definitions for the duration, one (τ_{DS}) based on the double support period as determined from ground reaction forces, and one (τ_{work}) based on the intervals of COM work performed by the two legs (Doke et al., 2005; Donelan et al., 2002a). It will be shown that all three definitions serve well in experimental comparisons; for brevity, only results for τ_{vel} are reported here, with other results reported in the Appendix.

Data analysis

We tested the model using least-squares fits to the predictions. The model predicts a series of trends with unknown coefficients *C* and *D* to be determined by each fit, with a different subscript for each prediction. Although the simplest walking models do not require an offset *D*, other models that include human-like mass or arc-shaped feet (Adamczyk et al., 2006) do predict an offset. Pre-transition COM velocity was tested with a model-based fit to Eqn 1:

$$v_{pre} = C_v \bar{v} + D_v\,, \tag{6}$$

applied to all walking conditions except the set in which walking velocity was held constant (CV; see Fig. 3). Redirection of COM velocity was tested with fits based on the actual angular change in velocity and on the impulses produced by the individual legs, from Eqn 2:

$$\delta_{vel} = C_{vel}\, s + D_{vel}\,, \tag{7}$$

$$\delta_{imp} = C_{imp}\, s + D_{imp}\,, \tag{8}$$

applied to all walking conditions except the set in which step length was held constant (CS; see Fig. 3).

Step-to-step transition work was tested with several model-based fits. These included model-based fits to velocity-based predictions of Eqn 4 for both collision (CO) and push-off (PO):

$$W_{CO} = C_{CO}\,(v_{pre}\cdot\delta_{vel})^2 + D_{CO}\,, \tag{9}$$

$$W_{PO} = C_{PO}\,(v_{pre}\cdot\delta_{vel})^2 + D_{PO}\,. \tag{10}$$

Similar model-based fits were applied to the simpler predictions of Eqn 5:

$$W_{CO} = C'_{CO}\,(\bar{v}\cdot s)^2 + D'_{CO}\,, \tag{11}$$

$$W_{PO} = C'_{PO}\,(\bar{v}\cdot s)^2 + D'_{PO}\,, \tag{12}$$

where the prime symbols (C' and D') indicate the use of speed and step length as gait-based predictors. These fits were applied to all data except the condition with highest step length (1.6*s**; see Fig. 3),

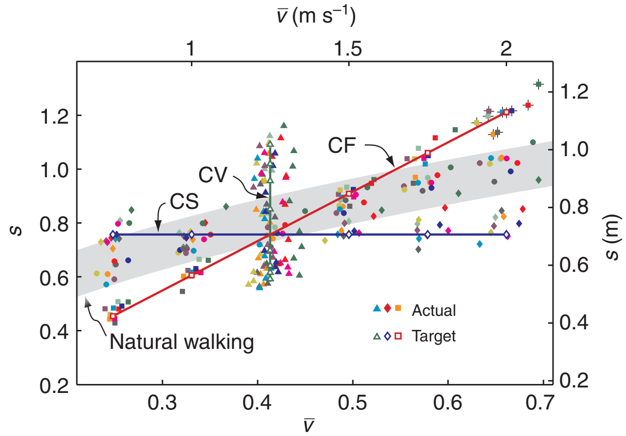

Fig. 3. Walking speeds and step lengths measured in four sets of conditions (24 total conditions). Each condition was specified as a target speed and step length (open symbols) that subjects attempted to match (actual combinations achieved are shown with filled symbols) while walking over ground. Sets included natural walking (circles) across a range of speeds using subjects' preferred step lengths; walking at constant step length (CS; diamonds); walking at constant step frequency (CF; squares); and walking at a constant speed over a range of step lengths (CV; triangles). The highest step length specified in the CF condition set was difficult for subjects to sustain without running (Donelan et al., 2002a); these data were excluded from model fits to COM work (trials marked with plus signs). Walking speed and step length were non-dimensionalized using leg length, L, and gravitational acceleration, g, as base units; equivalent dimensional mean values are shown on the top and right axes. N=10 subjects.

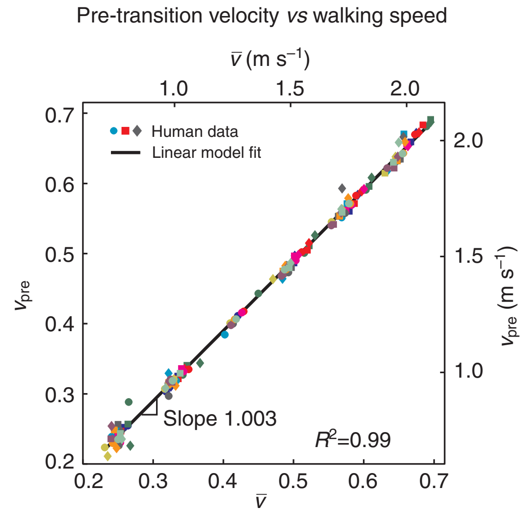

Fig. 4. Pre-transition center of mass (COM) velocity *vs* walking speed (v_{pre} *vs* $\bar{v}$). Data points shown are for all subjects (N=10) walking in natural walking, constant step frequency and constant step length conditions (constant speed conditions are excluded). Model-based fit shows that pre-transition COM velocity increased approximately linearly with walking speed (as predicted by Eqn 6), R^2=0.99. Walking speed can therefore predict the magnitude of COM velocity at initiation of the step-to-step transition. Velocities are shown in dimensionless units (left-hand and bottom axes) and m s^{-1} (right-hand and top axes).

which was deemed unsuitable because subjects were unable to sustain it for appreciable time (Donelan et al., 2002b).

We performed an additional set of model-based fits for the durations of the step-to-step transition. We performed linear regression fits of duration based on the three definitions for the transition (τ_{vel}, τ_{work}, τ_{DS}) to walking speed for the natural walking conditions. These are purely data-driven fits, because the simple walking models do not predict the duration of COM redirection.

We performed all regressions using dimensionless variables to account for differences in subjects' body size (Adamczyk et al., 2006). We used base units of subject mass, M, gravitational acceleration, g, and standing leg length, L. Velocity was therefore made dimensionless by the divisor $(gL)^{0.5}$, and work and energy by MgL. Step length was non-dimensionalized by leg length, L. Model fits are presented in dimensionless units, but SI units are also shown, using the mean non-dimensionalizing factor. For example, the mean non-dimensionalizing factor for work was MgL=630.6 J. We also accounted for inter-subject variations in kinematics and energetics by computing the offset D in each equation separately for each subject and then averaging these across subjects.

Statistical tests were performed on all fits to determine significant dependencies. We computed the 95% confidence interval from each fit, such that a confidence interval not including zero indicated significant change with α=0.05.

RESULTS

We found the kinematics and mechanics of the COM to behave approximately as predicted by simple walking models. Pre-transition COM velocity increased approximately linearly with walking speed. Both the angular redirection of COM velocity and the angular difference between leg impulses increased approximately linearly with step length. Work performed on the COM during the step-to-step transition also increased in rough proportion to the velocity-based and gait-parameter-based predictors. The duration of the step-to-step transition, however, appears to be longer than double support. Details of results are presented below.

Baseline values for the nominal walking condition were as follows. The nominal walking speed was an average of 1.27±0.01 m s^{-1} (mean ± s.d.), or 0.420±0.013 in dimensionless units. The preferred step length, s^*, averaged 0.714±0.030 m, or dimensionless 0.766±0.039. The directional change, δ_{vel}, in COM velocity was 0.324±0.053 rad, or 18.5±3.0 deg.; the angle, δ_{imp}, between leg impulses was 0.328±0.028 rad, or 18.8±1.6 deg. Mean pre-transition COM velocity, $\bar{v}$, was 1.23±0.02 m s^{-1}, or dimensionless 0.408±0.014. Negative collision work, W_{CO}, for each step of normal walking was 0.205±0.032 J kg^{-1}, or dimensionless 0.023±0.004. Positive push-off work, W_{PO}, performed on the COM during the step-to-step transition was 0.242±0.043 J kg^{-1}, or dimensionless 0.026±0.005.

Pre-transition COM velocity, v_{pre}, increased significantly with increasing walking speed (Fig. 4). Data were fit well (R^2=0.99) by the linear prediction of Eqn 6. Both δ_{vel} and δ_{imp} increased significantly with step length, as predicted by Eqns 7 and 8, across all conditions where step length was varied (Fig. 5). Dimensionless coefficients were C_v=1.003±0.009 (CI, 95% confidence interval) and D_v=0.011±0.006 for pre-transition velocity, C_{vel}=0.307±0.037 (CI) and D_{vel}=0.088±0.038 for velocity change angle (R^2=0.68; Fig. 5A), and C_{imp}=0.477±0.021 and D_{imp}=–0.037±0.022 for impulse angle (R^2=0.92; Fig. 5B).

The amount of work performed on the COM during the step-to-step transition also increased significantly with predicted quantities (Fig. 6). In terms of velocity-based predictors (Fig. 6A), W_{CO} increased approximately linearly as with Eqn 9 (R^2=0.74). W_{PO} increased approximately as with Eqn 10 (R^2=0.59). In terms of speed

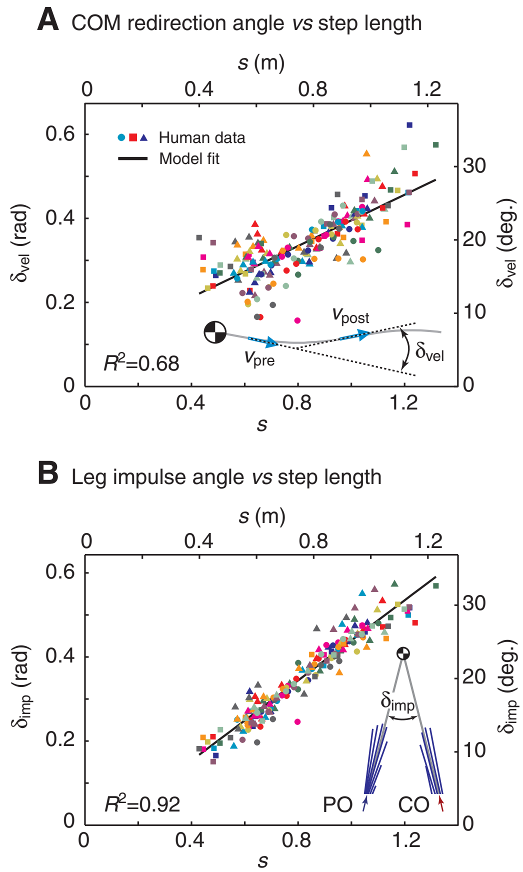

Fig. 5. Angular differences in the directions of (A) center of mass (COM) velocity at beginning and end of the step-to-step transition, and (B) trailing leg and leading leg ground reaction impulses during the step-to-step transition, both as a function of step length. Data shown are for all subjects (N=10) walking in the natural walking, constant step frequency and constant walking speed conditions, where step length was varied over a range of about 0.2 to 1.2 m (constant step length conditions are excluded). Model-based fits (Eqns 7 and 8) indicate that both angular differences increased approximately linearly with step length. Insets show definitions for angular change (δ_{vel}) in COM velocity and for angular difference (δ_{imp}) between ground reaction impulses performed by each leg, push-off (PO) and collision (CO). Step lengths are shown normalized by leg length, L (bottom axes), as well as in units of meters (top axes).

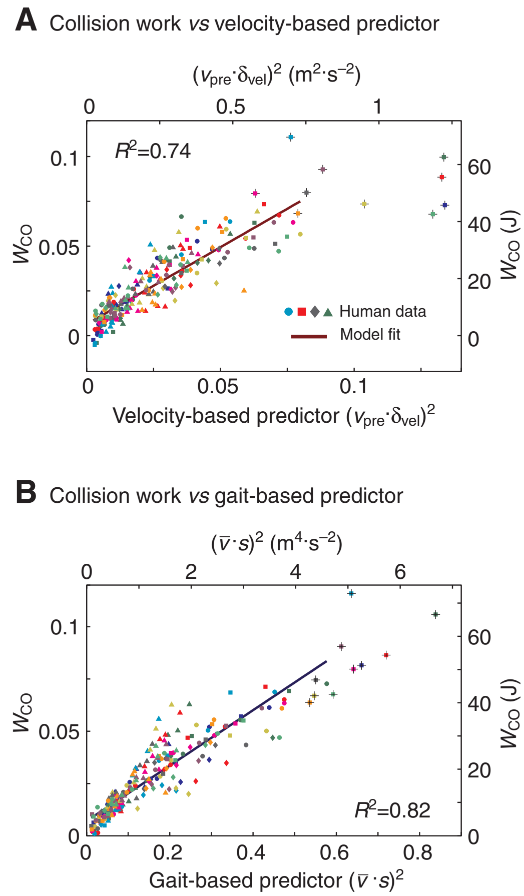

Fig. 6. Negative center of mass (COM) work, W_{CO}, performed by the leading leg during the step-to-step transition *vs* model predictions. Data shown are for all subjects (N=10) walking in all 24 experimental conditions. (A) The simple walking model predicts (Eqn 9) that work per step to redirect the COM will increase with $(v_{pre}\cdot\delta_{vel})^2$, where v_{pre} is the magnitude of COM velocity at initiation of the step-to-step transition, and δ_{vel} is the angular change the COM velocity undergoes during the transition (see Fig. 5A). Data matched the model (solid line) reasonably well (R^2=0.74). (B) A simpler predictor is the squared product of walking speed and step length ($\bar{v}\cdot s$), derived from predicted linear relationships (Eqn 11). Data matched the model (solid line) reasonably well (R^2=0.82). The COM negative work required for gait is well predicted by the trends derived from our simple dynamic walking model. Trials with the highest speeds and longest steps (marked by plus symbols) were excluded from model-based fits because subjects could not consistently maintain those gaits without running (see Fig. 3). For both A and B, two sets of axes are shown, with left-hand and bottom axes in dimensionless units, and right-hand and top axes in dimensional units.

and step length as gait-parameter-based predictors (Fig. 6B), W_{CO} increased approximately linearly as in Eqn 11 (R^2=0.82). W_{PO} also increased approximately as in Eqn 12 (R^2=0.65). The dimensionless coefficients using velocity-based predictors were C_{CO}=0.804±0.068 (CI), D_{CO}=0.009±0.004, C_{PO}=0.507±0.064 and D_{PO}=0.017±0.004. Coefficients using speed and step length were C'_{CO}=0.128±0.008 (CI) and D'_{CO}=0.008±0.003, C'_{PO}=0.081±0.009 and D'_{PO}=0.017±0.003.

The duration of the step-to-step transition was dependent on whether it was defined based on GRF, velocity or work. The COM hodograph showed that the velocity and work-based definitions consistently exceeded double support (measured by the GRFs) as a function of walking speed (Fig. 7A). The fraction of a stride, τ_{DS}, spent in double support decreased with speed, but the durations based on angular change in COM velocity and on COM work were both greater and relatively constant (Fig. 7B). At the nominal speed of 1.25 m s^{-1}, double support was about 14% of a stride, whereas τ_{vel} was 27%, and τ_{work} was 20%. Double support decreased with increasing walking speed, from about 17% to 9% of a stride cycle, with dimensionless slope of –0.36±0.04 (CI) and offset of 0.42±0.02 (R^2=0.82). The other durations, for velocity change and work performed on the COM, both increased by very slight amounts. τ_{vel} ranged from 27% to 28% of a stride, with slope 0.09±0.09 (CI) and offset 0.36±0.04 (R^2=0.08). τ_{work} ranged from about 19% to 21% of a stride, with slope 0.09±0.08 (CI) and offset 0.51±0.04 (R^2=0.07). (The latter two R^2 values were low due to the near-zero slopes.)

DISCUSSION

Simple models of human walking predict how the COM will be redirected, assuming each leg is fairly rigid during its stance phase. These models have predicted trends in overall mechanical work and

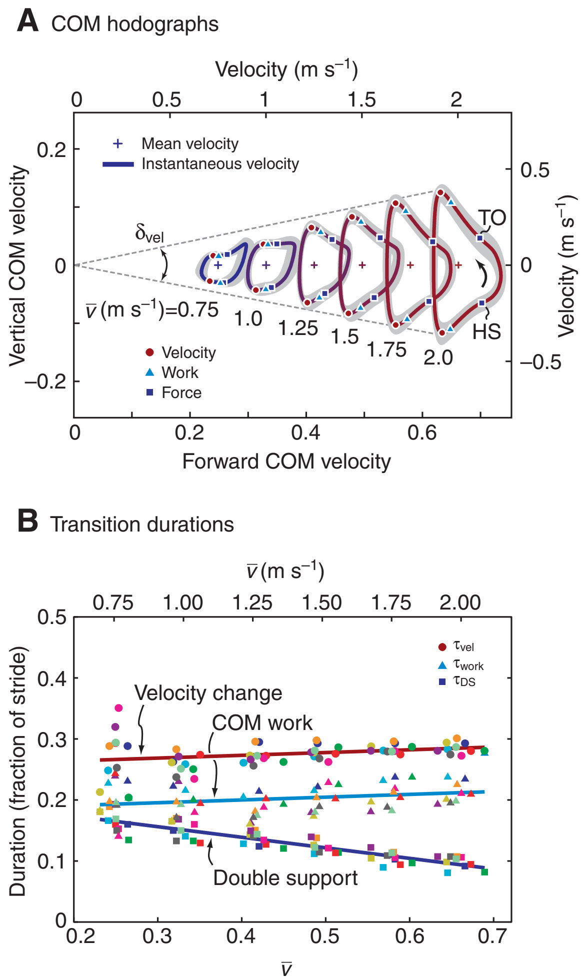

Fig. 7. (A) Plot of center of mass velocity components (referred to as COM hodographs) for a variety of walking speeds, showing sagittal plane velocity fluctuations. Averaged velocity data is shown for all subjects (N=10) in natural walking conditions, with standard deviations denoted by shaded areas, and mean walking speed denoted by small filled circles. Hodographs show maximum angular redirection δ_{vel} of COM velocity (broken lines for 2 m s^{-1} condition), as well as the pre- and post-transition velocities at the points (small circles) where COM velocity is directed most downward and most upward. The beginning of push-off work and end of collision (small triangles) occur over slightly shorter durations, and the double support period between heel-strike and toe-off (HS and TO; small squares) occurs over a still shorter duration. Both angular redirection and pre-transition velocity increase consistently with walking speed, requiring greater work to redirect COM velocity. (B) Durations of step-to-step transition in natural walking (in terms of fraction of a stride) as a function of walking speed. Velocity change refers to the duration (τ_{vel}) of the angular redirection of COM; COM work refers to the duration (τ_{work}) over which the trailing leg performs positive work and the leading leg performs negative work in the time interval surrounding double support. Double support refers to the period (τ_{DS}) when both legs contact the ground.

metabolic energy rates with changes in speed, step length and step width (Donelan et al., 2001; Donelan et al., 2002a; Donelan et al., 2002b), but they have not previously been tested in terms of the actual motion of the COM. The present study investigated whether the rigid leg assumption applies sufficiently to predict the angular redirection of the COM and the associated work performed on the COM by the individual legs. We found that, across a wide range of walking conditions, human subjects appear to redirect the COM largely as the models predict. For example, mechanical quantities such as the work performed on the COM during the step-to-step transition change largely as a function of only walking speed and step length. These same models do not, however, predict the duration of the transition, which was observed to exceed double support and therefore indicate non-rigid stance leg behavior.

The amount of COM redirection appears to be determined mainly by walking speed and step length. These two gait parameters influence, respectively, the magnitude and direction that describe the COM velocity. In both model and human, the step-to-step transition occurs at greater COM velocity than the inverted pendulum phase (see Fig. 1B,D). Between these phases, at initiation of the step-to-step transition, the velocity magnitude v_{pre} was found empirically to nearly equal walking speed (Fig. 4), even for step lengths and frequencies differing greatly from the nominal walking condition. The close model fit (R^2=0.99) demonstrates that pre-transition velocity magnitude is determined almost entirely by walking speed and much less by step length or frequency. By contrast, δ_{vel} is determined primarily by step length (Fig. 5). This direction change can be attributed to the impulses applied along the legs, with angle δ_{imp} also increasing with step length. However, δ_{vel} depends not only on leg impulses but also on the effect of gravity and the displacement of the COM during the step-to-step transition. These factors cause δ_{vel} to increase less than δ_{imp} with increasing step length, although the relative amount cannot be predicted by our simple model, which incorporates neither duration nor displacement in the step-to-step transition. These effects partially account for the observed poorer model fit for δ_{vel} than for δ_{imp} (R^2=0.68 *vs* 0.92, respectively). The model cannot predict what additional aspects of gait contribute to these quantities, but it is clear that step length is a major determinant of COM velocity direction change, and walking speed is a major determinant of the COM velocity at the beginning of the step-to-step transition.

COM redirection requires that work be performed on the COM. We found step-to-step transition work to increase similarly with both velocity- and gait-parameter-based predictors (see Fig. 6). The predictions of Eqns 4 and 5 again assume a perfectly impulsive step-to-step transition, which differs from reality. Collision work per step increased somewhat more than push-off work (C_{CO}>C_{PO} for both velocity- and gait-parameter-based predictors), perhaps because positive work by muscles is more strictly rate-limited than negative work, which can be performed in part by passive tissues. The push-off limitation means that, at higher speeds and step lengths, some of the work to offset the collision must be performed at other parts of the stride (e.g. Kuo et al., 2005). It is also interesting to note that the gait-based predictors – speed and step length – yielded slightly better fits than the velocity-based predictors (e.g. R^2=0.82 *vs* 0.74 in Fig. 6). The errors of multiple predictions (Fig. 2B) could conceivably have accumulated to yield a very poor fit overall, but this was evidently not the case. Closer inspection reveals that the fits are poorer for very long steps (over 1 m) and fast speeds (about 2 m s^{-1}), combinations not normally employed by humans in walking. Others have suggested that pendular mechanics do not apply at such step lengths (Bertram et al., 2002). Data throughout the range of conditions appear to exhibit slight nonlinearity, perhaps with an exponent lower than the square of the product of speed and step length. An arbitrary nonlinear regression would almost surely fit better than the model's prediction, but would lack explanation. The simple model, while imperfect, provides both a reasonable fit and a mechanistic explanation.

Our results are consistent with previous reports of COM motion. These include reports of δ_{vel} for a nominal gait (Adamczyk et al., 2006), of COM work for walking at varying step lengths but fixed step frequencies (Donelan et al., 2002a) and of COM work at varying step widths but fixed step lengths and frequencies (Donelan et al., 2001). The present study shows that even when gait parameters are varied considerably, step length and walking speed largely account for the angular redirection of the COM, the pre-transition magnitude of COM velocity and the work performed on the COM during the step-to-step transition. Similar pendulum mechanics are thus preserved across a wide range of gait parameters, as is assumed in a number of theoretical studies of walking (Kuo, 2001; Srinivasan and Ruina, 2006; Usherwood et al., 2008). One such model (Ruina et al., 2005) suggested that COM redirection work could be very different if humans somehow alter the relative timing of GRFs associated with push-off and collision. The empirical results here show quite consistent timing (Fig. 7) and work (Fig. 6) of the step-to-step transition. We also computed an additional metric to estimate the overlap of push-off and collision, as proposed by Ruina et al. (Ruina et al., 2005), and found push-off to always precede collision with similar functional overlap (see Appendix for details). Even though the duration of double support changes with speed, the work of COM redirection is performed in a very consistent manner across a wide range of walking patterns.

Consistent COM motion also has implications for energy expenditure. Metabolic energy expenditure has been found to increase linearly with COM work, as a function of step width or length (Donelan et al., 2001; Donelan et al., 2002a), when step frequency is kept fixed. This controls for possible energetic costs, such as for moving the legs back and forth, that may increase with step frequency (Doke et al., 2005). We observed consistent COM mechanics not only for a wide range of step frequencies but also during natural walking, when subjects walked at preferred (unconstrained) step frequency. The present results suggest that COM work may effectively quantify the contribution of step-to-step transitions to energetic cost, for both natural and constrained walking.

Two key outcome variables in this study – magnitude and direction of COM velocity – may be visualized from the COM hodograph plot (Fig. 1; Fig. 7A). A plot of vertical *vs* forward COM velocity yields a closed, counter-clockwise curve for each step, facilitating comparison of gait features relevant to the step-to-step transition. The left-most portion corresponds to the inverted pendulum phase, as the COM rises and slows prior to mid-stance, and then speeds up while falling forward. The right-most portion corresponds to the step-to-step transition, as the COM is accelerated and redirected upward by the trailing leg's push-off, and is decelerated and redirected upward by the leading leg's collision. The COM hodographs for the natural walking conditions (Fig. 7A) show that the pre-transition velocity and angular redirection both change consistently with speed. It is notable that, as walking speed increases, a greater proportion of the redirection occurs before and after double support. This is confirmed by the decreasing duration of double support as a function of speed, relative to the duration of actual velocity change (Fig. 7B). It is also notable that the velocity profile during the step-to-step transition is quite consistent across subjects, yet also exhibits a different stereotypical pattern for each speed. This consistency among healthy humans may also make the COM hodograph helpful for examining abnormal or impaired gaits. It might, for example, help determine whether a gait impairment has most effect on the inverted pendulum phase or the step-to-step transition.

Our analysis emphasizes the step-to-step transition as a key issue in pendulum-like walking. This contrasts with previous studies (e.g. Cavagna and Kaneko, 1977; Willems et al., 1995) that quantified fluctuations of kinetic and potential energy but could not predict their dependence on gait parameters. Energy fluctuations are surely important indicators of pendulum mechanics but cannot quantify work performed to redirect the COM velocity between pendulum-like steps. Previous studies have also plotted the COM trajectory in space as a closed path (Margaria, 1976), but again with little predictive ability. Our results show that trends in both COM work and velocity changes can be predicted as a function of gait parameters while also remaining compatible with pendulum mechanics.

Some important gait features cannot be examined using the simple model presented here. The model's rigid legs perform a perfectly impulsive step-to-step transition and therefore can predict neither the duration nor the actual trajectory of COM displacement as it is redirected. The duration of the step-to-step transition, as defined by angular redirection or COM work (Fig. 7), appears considerably longer than that of double support. We originally used COM work to indicate the simultaneous positive and negative work performed during double support (Donelan et al., 2002b) but later found push-off work to begin earlier and collision work to end later than that period (Donelan et al., 2002a). The model performs minimum work by pushing off impulsively just prior to collision whereas humans cannot produce ideal impulses and must perform more work over finite time. The model cannot predict how the trailing leg begins extending prior to double support or how the leading leg stops flexing after double support. Our results here show that even though double support captures much of the step-to-step transition and its dependence on speed and step length, it also underestimates the total angular change in COM velocity by about 19%, and the associated work by about 25% at the nominal walking speed. This behavior also partially explains why the model predicts trends in work but not the actual amounts, equivalent to the coefficients of the model fits (Eqns 6–12). The additional features necessary to model these effects may include axial leg compliance as has already been incorporated in other studies (e.g. Alexander, 1991; Buczek et al., 2006; Geyer et al., 2006).

The variables studied here appear to be physiologically relevant. We previously hypothesized that work performed to redirect the COM velocity may explain a portion of the metabolic energy cost of walking. We have found COM work to predict metabolic cost under controlled conditions for variables such as step length (Donelan et al., 2002a), step width (Donelan et al., 2001) and the shape of rigid foot bottoms (Adamczyk et al., 2006). The present study shows that a simple dynamic walking model, despite its limitations, remains useful for predicting trends in COM velocity and work. The trends may be predicted largely by simple gait parameters, such as step length and step frequency, that can be controlled experimentally. Assuming these controls do not induce abnormal gait, they make it possible to control for step-to-step transition costs and thereby examine other contributors to the overall energetic demands of normal walking. Finally, we also speculate that the observed duration of the step-to-step transition may be physiologically advantageous. It allows the work for COM redirection to occur over an extended time, possibly reducing the peak force and power requirements of muscles.

Conclusions

Human legs are not perfectly rigid during either the inverted pendulum phase or the step-to-step transition of walking. Despite these deviations from model assumptions, changes in magnitude and direction of COM velocity are predicted well by dynamic walking models. Greater walking speeds lead to greater COM velocity magnitude, and greater step lengths lead to greater redirection angle. These variables in turn predict work performed on the COM – a major contributor to metabolic energy expenditure – as a function of walking speed and step length. There are also substantial limitations to rigid-legged models, namely their inability to predict the duration and displacement of the step-to-step transition. Empirical observations show that the duration exceeds double support, with the non-rigid single support leg contributing substantially to the step-to-step transition.

APPENDIX

Step-to-step transition durations

We examined two other definitions for the duration of the step-to-step transition in addition to that based on center of mass (COM) velocity, τ_{vel} (see Fig. A1). The first, τ_{DS}, was based on the double support period as determined from the vertical ground reaction force (Donelan, 2002b). The second, τ_{work}, was based on the intervals over which the trailing and leading legs perform work on the COM (see Fig. A1B) (Donelan, 2002a). Here we compare the least-squares fit between model and data for all three of these definitions (see Table A1).

We found the overall trends predicted by the model to be largely independent of the particular definition used. All definitions produced fits that indicated approximately linearly increasing trends. Those based on COM velocity and work rate, however, agree better with each other and with data, yielding R^2 values ranging 0.60 to 0.99. The definition based on vertical GRF yielded similar trends but somewhat poorer fits, with R^2 values as low as 0.31. This is because the fraction of a stride spent in double support decreases at faster step frequencies or speeds, so that double support captures less of the total COM velocity redirection δ_{vel} (see Fig. 7) and COM work (Fig. A1).

Previous studies have also reported that COM redirection occurs over more than double support alone (Donelan et al., 2002a). Push-off positive work begins prior to opposite leg heel strike, and collision negative work ends after opposite leg toe-off. This causes estimates based on double support to underestimate the work, despite successfully capturing the broad trends of work increasing with step length. Both COM velocity and COM work are better delineators of the step-to-step transition.

Relative timing of push-off and collision

Ruina et al. presented a theoretical model of the step-to-step transition in which the relative overlap of the push-off and collision impulses can be varied within their infinitesimal duration [see Appendix in Ruina et al. (Ruina et al., 2005)]. They defined an 'overlap parameter' (s_o) to represent how simultaneously the push-off and collision impulses occur. We computed an adapted form of s_o for humans, modified to account for transitions of finite duration. We defined each leg's net impulse (P^*_{PO}, P^*_{CO}) as the time integral of its three-dimensional GRF during the whole step-to-step transition. A parameter $q_{PO}(t)$ quantifies the fraction of the net push-off impulse that has been completed prior to time t. We defined this push-off fraction as the projection of the completed push-off impulse, $P_{PO}(t)$, along the net impulse direction vector, $\hat{P}^*_{PO}$, normalized by the magnitude of the net impulse P^*_{PO}:

$$q_{PO}(t) = \frac{P_{PO}(t) \cdot \hat{P}^*_{PO}}{\left\| P^*_{PO} \right\|} . \quad \text{(A1)}$$

We defined $q_{CO}(t)$ similarly with respect to the collision impulse P^*_{CO}. The overlap parameter s_o is defined by the integral of the collision fraction with respect to the push-off fraction:

$$s_o = \int_0^1 q_{CO} \mathrm{d}q_{PO} . \quad \text{(A2)}$$

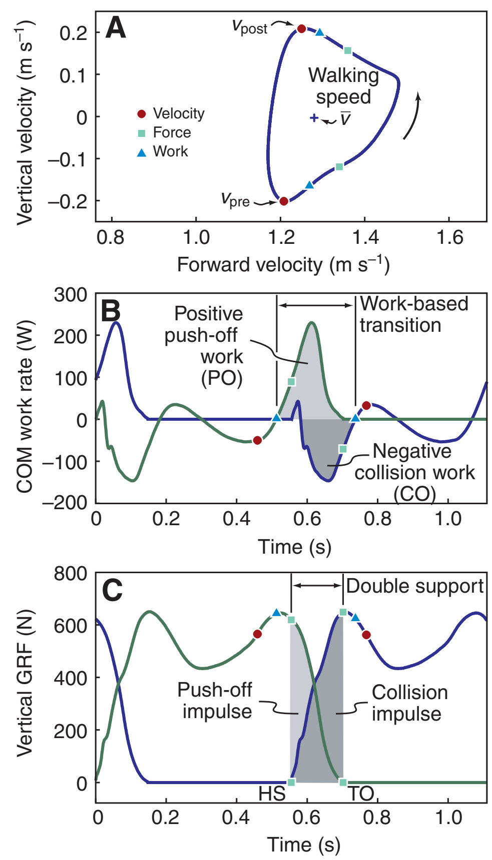

Fig. A1. Three possible definitions for duration of the step-to-step transition, based on (A) center of mass (COM) velocity, (B) COM work and (C) vertical ground reaction forces (GRF), illustrated with a typical subject's nominal gait at 1.25 m s^{-1}. (A) Sagittal plane COM hodograph shows trajectory of vertical and forward components of COM velocity. The step-to-step transition may be alternatively defined based on the maximum angular excursion of COM velocity (τ_{vel}; marked by circles), by intervals of positive and negative work performed on the COM (τ_{work}; marked by triangles) and by the period of double support from GRFs (τ_{DS}; marked by squares). (B) The trailing leg performs positive push-off work on the COM, starting slightly before double support and ending with toe-off. The leading leg performs negative collision work, starting with heel-strike and ending slightly after double support ends. These intervals of positive and negative work were used to define a work-based step-to-step transition (shaded regions), with the alternative definitions yielding different intervals for computing work (circles and squares). (C) The double support period served as the third definition for the step-to-step transition. Ground reaction forces were integrated during the transition to determine the impulse in each leg, for push-off and collision (shaded regions, showing vertical component only; HS denotes heel-strike and TO denotes toe-off). Each definition is based on a different signal, but COM velocity and COM work both indicate that the step-to-step transition occurs over a duration exceeding double support.

The overlap parameter can vary from 0 (push-off entirely before collision, as assumed for the model in the present study) to 1 (collision entirely before push-off), with 0.5 indicating simultaneous impulses.

Subjects in the current study demonstrated very consistent values of s_o across all conditions. Using step-to-step transitions based on COM velocity, s_o was 0.047±0.026 (mean ± s.d.), indicating that the push-off impulse occurred considerably earlier than the collision impulse. There were slight, statistically significant decreasing trends in s_o with increases in speed, step length and step frequency. The

Table A1. Comparison of three different signals for defining the step-to-step transition, applied to seven model prediction equations

Equation (equation number in parentheses)	Signal	Slope C	Offset D	R^2
$v_{pre}=C_v\bar{v}+D_v$ (6)	COM velocity	1.003±0.009	0.011±0.006	0.99
	COM work rate	0.998±0.007	0.005±0.005	0.99
	Vertical GRF	1.094±0.018	–0.017±0.012	0.98
$\delta_{vel}=C_{vel}s+D_{vel}$ (7)	COM velocity	0.307±0.037	0.088±0.038	0.69
	COM work rate	0.259±0.025	0.069±0.026	0.75
	Vertical GRF	–0.001±0.038	0.217±0.039	0.31
$\delta_{imp}=C_{imp}s+D_{imp}$ (8)	COM velocity	0.477±0.021	–0.037±0.022	0.91
	COM work rate	0.470±0.019	0.008±0.020	0.93
	Vertical GRF	0.609±0.024	–0.067±0.025	0.93
$W_{CO}=C_{CO}(v_{pre}\cdot\delta_{vel})^2+D_{CO}$ (9)	COM velocity	0.804±0.068	0.009±0.004	0.75
	COM work rate	1.153±0.088	0.008±0.004	0.77
	Vertical GRF	1.388±0.181	0.009±0.004	0.53
$W_{PO}=C_{PO}(v_{pre}\cdot\delta_{vel})^2+D_{PO}$ (10)	COM velocity	0.507±0.064	0.017±0.004	0.62
	COM work rate	0.740±0.089	0.017±0.004	0.60
	Vertical GRF	0.688±0.147	0.019±0.003	0.40
$W_{CO}=C'_{CO}(\bar{v}\cdot s)^2+D'_{CO}$ (11)	COM velocity	0.128±0.008	0.008±0.003	0.82
	COM work rate	0.129±0.008	0.008±0.003	0.81
	Vertical GRF	0.086±0.009	0.010±0.003	0.65
$W_{PO}=C'_{PO}(\bar{v}\cdot s)^2+D'_{PO}$ (12)	COM velocity	0.081±0.009	0.017±0.003	0.66
	COM work rate	0.082±0.009	0.017±0.004	0.63
	Vertical GRF	0.046±0.008	0.019±0.003	0.49

Signals were based on center of mass (COM) velocity, the interval over which positive and negative work is performed on the COM, and vertical ground reaction force (GRF) indicating double support. The best-fit coefficients (C and D) are reported, along with R^2 values.

strongest trend was for speed and had a best linear fit of s_o=–0.124$\bar{v}$+0.101, R^2=0.49. Other step-to-step transition definitions captured shorter durations, which led to higher values of s_o. This change was expected, because restricted step-to-step transition timing cuts off the beginning of push-off and the end of collision. For timing based on COM work rate, s_o was 0.080±0.039 (mean ± s.d.); for timing based on vertical GRF, s_o was 0.151±0.027. Trend lines were similar with timing based on COM work rate; however, trends differed for timing based on vertical GRF, showing that s_o decreased for increasing step frequency, increased with increasing step length and had no trend with respect to speed. Regardless of the timing basis, s_o was always less than 0.24, indicating that the push-off impulse always occurred considerably earlier than the collision impulse.

LIST OF SYMBOLS AND ABBREVIATIONS

C, C'	scaling coefficients of linear model fits to data
CF	constant step frequency
COM	center of mass
CS	constant step length
CV	constant speed
D, D'	offset coefficients of linear model fits to data
f^*	preferred step frequency at designated walking speed $\bar{v}^*$
GRF	ground reaction force
L	leg length
M	body mass
P^*_{CO}	net collision impulse
$\hat{P}^*_{CO}$	direction vector of the net collision impulse
$P_{CO}(t)$	cumulative collision impulse that has occurred before time t within the step-to-step transition
P^*_{PO}	net push-off impulse
$\hat{P}^*_{PO}$	direction vector of the net push-off impulse
$P_{PO}(t)$	cumulative push-off impulse that has occurred before time t within the step-to-step transition
$q_{CO}(t)$	cumulative fraction of the net collision impulse completed prior to time t within the step-to-step transition
$q_{PO}(t)$	cumulative fraction of the net push-off impulse completed prior to time t within the step-to-step transition
s	step length
s_o	overlap parameter for push-off and collision impulses
s^*	preferred step length at designated walking speed $\bar{v}^*$
$\bar{v}$	mean walking speed
$\bar{v}^*$	designated nominal walking speed 1.25 m s^{-1}
v_{mid}	COM velocity at the middle of the model's step-to-step transition, between the push-off and heel-strike impulses
v_{post}	COM velocity at the end of the step-to-step transition
v_{pre}	COM velocity at the beginning of the step-to-step transition
W	work
W_{CO}	collision negative work
W_{PO}	push-off positive work
δ	directional change in velocity in simple model
δ_{imp}	angular change in ground reaction impulses during step-to-step transition
δ_{vel}	angular change in center-of-mass velocity during step-to-step transition
g	gravitational acceleration
τ_{DS}	duration of step-to-step transition based on the double support period as determined from ground reaction forces
τ_{vel}	duration of step-to-step transition based on COM velocity changes
τ_{work}	duration of step-to-step transition on the intervals of COM work for push-off and collision

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